Problems Solved:

46	47	48	49	50

Name:

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Problem 46. Does there exist for every finite language $L \subseteq \{0,1\}^*$ a Turing machine D such that D

1. takes as input the code $\langle M \rangle$ of a Turing machine M that stops on every input and
2. decides whether $L \subseteq L(M)$ holds?

Justify your answer.

Problem 47. Let $L = \{1^n 0 1^m \mid m, n \in \mathbb{N} \wedge n > 0 \wedge m > n\}$.

- Describe (informally) a Turing machine M with $L(M) = L$.
- Analyse the worst-case time and space complexity of M in terms of the length l of the input word.

Problem 48. Take the following recursive program.

```
1 f(n,b) ==
2     if n < 1 then return 0
3     d := floor(n/3)
4     return b + f(d,1) + 2*f(d,2)
```

Let $C(n)$ be the number of comparisons executed in line 2 while running $f(n, 0)$ for some positive integer n.

1. Write down a recurrence for C and determine enough initial values.
2. Solve that recurrence for the given initial values and arguments n of the form $n = 3^m$.
3. Prove by induction that your solution is correct.

Problem 49. Consider a RAM program that evaluates the value of $n! = \prod_{i=1}^{n} i$ in the naive way (by iteration). Analyze the worst-case asymptotic time and space complexity of this algorithm on a RAM assuming the existence of operations operation `ADD r` and `MUL r` for the addition and multiplication of the accumulator with the content of register r.

1. Determine a Θ-expression for the number $S(n)$ of registers used in the program with input n (space complexity).
2. Determine a Θ-expression for the number $T(n)$ of instructions executed for input n (time complexity in constant cost model),

3. Determine an O-expression for the asymptotic time complexity $C(n)$ of the algorithm for input n assuming the logarithmic cost model for a RAM (with cost $al \cdot rl$ for operation `MUL r` where al is the digit length of the content of the accumulator and rl is the digit length of the content of register r).

   Hint: approximate in the asymptotic analysis summation $\sum_{i=a}^{b} T$ by integration $\int_{a}^{b} T \mathrm{d}i$ and solve this integral; you may use a computer algebra system or WolframAlpha for this purpose.

Problem 50. Consider the program

```
static void permutations(char[] a, int i)
  {
    if (i >= a.length)
    {
      System.out.println(new String(a));
      return;
    }
    for (int j=i; j<a.length; j++)
    {
      swap(a, i, j);
      permutations(a, i+1);
      swap(a, i, j);
    }
  }
```

where a call `permutations(s.toCharArray(), 0)` prints all permutations of string s. The function call `swap(a, i, j)` exchanges the i-th and j-th entries of the array a.

1. Draw a recursion tree for a call `permutations(<abcd>,0)` (where `<abcd>` denotes the character array with elements a, b, c, d). What is the height of the tree? What is the number of nodes in every level of the tree? What is the total number of nodes in this tree?

2. Give a recurrence for the total number $S(n)$ of calls of `swap` in an execution of `permutations(a,i)` where $n =$ `a.length-i`.

3. Give a summation formula for $S(n)$ by adding the number of calls in every level of the recursion tree.

4. Give a complexity estimation $S(n) = O(T(n))$ for some closed formula n (justify your estimation semi-formally).