Problems Solved:

16	17	18	19	20

Name:

Matrikel-Nr.:

Problem 16. Answer the following questions and provide reasons for your answers. You need to be concerned about the implications for the general computability of functions.

1. Is it possible to encode the source code of a RAM program into a sequence of natural numbers?
2. Is it possible to construct a RAM program U, which can simulate the work of any other RAM program?
3. Is it possible to construct a RAM program R that uses only the first n registers ($n \in \mathbb{N}$ sufficiently big, but fixed), such that R can simulate the work of any other RAM program?

Remark: Note that if you answer simply with yes-no, without providing any reasonable arguments, your solution will not be graded positively.

Problem 17. Let $\Sigma = \{a, b\}$. We encode a and b on the input tape of a RAM by 1 and 2 and a word $w \in \Sigma^*$ by a respective sequence of 1's and 2's.
We say that a RAM R *accepts* a word $w \in \Sigma^*$ if R starts with the coded word w on its input tape and terminates after having written a non-zero number on its output tape. We define $L(R) := \{w \in \Sigma^* \mid R \text{ accepts } w\}$.
Let F be a RAM that terminates for every input and whose program does not contain "loops", i.e., each instruction is executed at most once.
Derive answers for the following questions. (Give ample justifications, just saying 'yes' or 'no' is not enough.)

1. Is $L(F)$ as a language over Σ finite?
2. Is $L(F)$ as a language over Σ regular?

Problem 18. In the following use *only* the definition of a *loop program* as given in Def. 23 of the lecture notes, Section 3.2.2. Note that it is not allowed to use abbreviations like

```
xi := xj − xk;
xi := xj + xk;
```

Furthermore, the variables in a loop program are only $x_0, x_1, \ldots$

1. Show that the function

$$s(x_1, x_2) = \begin{cases} 1 & \text{if } x_1 < x_2, \\ 0 & \text{otherwise} \end{cases}$$

is loop computable. I.e. give an explicit loop program for s.

2. Write a loop program that computes the function $d : \mathbb{N}^2 \to \mathbb{N}$ where $d(x_1, x_2)$ is $k \in \mathbb{N}$ such that $k \cdot (x_2 + 1) = x_1 + 1$ if such a k exists. The result is $d(x_1, x_2) = 0$, if a k with the above property does not exist.

   For simplicity in the program for d, you are allowed to use a construction like the following (with the obvious semantics) where P is an arbitrary loop program.

   ```
   IF xi < xj THEN P END;
   ```

   Note: Only $<$ is allowed in the condition and there is no "ELSE" branch.

Problem 19. Provide a loop program that computes the function $f(n) = \sum_{k=1}^{n} k(k+1)$, and thus show that f is loop computable.
You are only allowed to use the constructs given in Definition 23 of the lecture notes.

Problem 20. Suppose P is a while-program that does not contain any `WHILE` statements, but might modify the values of the variables x_1 and x_2.
Transform the following program into Kleene's normal form.
Hint: first translate the program into a goto program, replace the `GOTO`s by assignments to a control variable, and add the `WHILE` wrapper.

```
x0 := 0
WHILE x1 DO
    x1 := x1 - 1;
    x2 := x1;
    WHILE x2 DO
        P;
    END;
END;
x0 := x0 + 1
```