Gruppe	Hemmecke (10:15)	Hemmecke (11:00)		Popov											
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Klausur 1

Berechenbarkeit und Komplexität

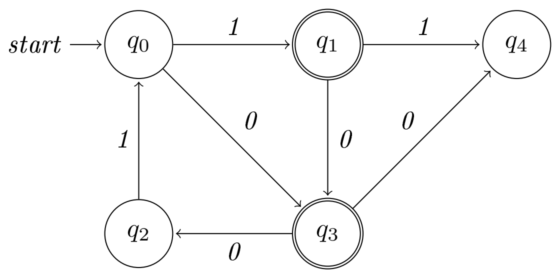
17. November 2017

Part 1 NFSM2017

Let N be the nondeterministic finite state machine

$$(\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \nu, \{q_0\}, \{q_1, q_3\}),$$

whose transition function ν is given below.



1		no

Is $1001100111 \in L(N)$*?*

Following the states it ends up in q_4.

2	yes	

Is $00110010 \in L(N)$*?*

Follow the states $q_0, q_3, q_2, q_0, q_1, q_3, q_2, q_0, q_3$.

3		no

Is $L(N)$ finite?

4	yes	

Does there exist a regular expression r such that $L(r) = \overline{L(N)} = \{0,1\}^* \setminus L(N)$?

$L(N)$ is regular and so is its complement.

5	yes	

Is $\overline{L(N)}$ recursively enumerable?

$L(N)$ is regular. Hence, $\overline{L(N)}$ is regular, and thus also recursively enumerable.

6	yes	

Is there a deterministic finite state machine M with less than 50 states such that $L(M) = L(N)$?

According to the subset construction, there must be a DFSM with at most $2^5 = 32$ states.

7	yes	

Is there an enumerator Turing machine G such that $Gen(G) = L(N)$?

8	yes	

Does there exists a deterministic finite state machine D such that $L(D) = L(N) \circ \overline{L(N)}$?

$L(N)$ and $\overline{L(N)}$ are both regular. Concatenation of two regular languages gives a regular language.

Part 2 Computable2017

Let M be a Turing machine such that it accepts a word, if and only if it is a tautonym. A tautonym is a word or a name made up of two identical parts, such as so so, tom tom, Baden Baden *or* Pago Pago.

9	yes		*Is $L(M)$ recursively enumerable?*
10	yes		*Is $L(M)$ recursive?*
11		no	*Is $L(M)$ finite?*

There can be arbitrarily large tautonyms.

12		no	*Let L be a recursively enumerable language. Can it be concluded that $L(M) \cap L$ is recursive?*

Intersection of recursive and recursively enumerable languages is recursively enumerable but not necessarily recursive.

13	yes		*Is every primitive recursive function also a μ-recursive function?*
14	yes		*Does there exist a μ-recursive function that is LOOP computable?*
15	yes		*Is every Turing-computable function $f : \{0,1\}^* \to \{0,1\}^*$ also μ-recursive (identifying sequences of $0, 1$ with natural numbers in binary representation)?*

Part 3 *Pumping2017*

Let

$$L_1 = \{\, a^{2n} b^{7m} a^{n-m+2017} \mid n, m \in \mathbb{N}, m < n < 2017 \,\} \subset \{a,b\}^*\,,$$
$$L_2 = \{\, a^m b^n a^{n+m+2017} \mid m, n \in \mathbb{N}, m > n > 1 \,\} \subset \{a,b\}^*\,.$$

16	yes		*Is there a deterministic finite state machine M such that $L(M) = L_1$?*

The language L_1 is finite and thus regular.

17		no	*Is there a deterministic finite state machine M' such that $L(M') = L_2$?*
18	yes		*Is there an enumerator Turing machine G such that $Gen(G) = L_2$?*
19	yes		*Is there a deterministic finite state machine D such that $L(D) = L_1 \cap L_2$?*

The language $L_1 \cap L_2$ is finite and thus regular.

20	yes		*Is there a language L such that $L \cup L_2$ is regular?*

Yes. Take as L the complement of L_2.

Part 4 *WhileLoop2017*

Take the WHILE program P defined as:

```
x0 := 0
while x1 <> x2 do
    x0 := x0 + 1;
    x1 := x1 + 1
end;
```

Remark: Here a loop

```
while xi <> xj do ...
```

has the intuitive meaning "iterate ... while the value of variable x_i is different from the value of x_j" (which can be expressed by a program in the core syntax of WHILE programs). Take also the WHILE program P' defined as (P is as above):

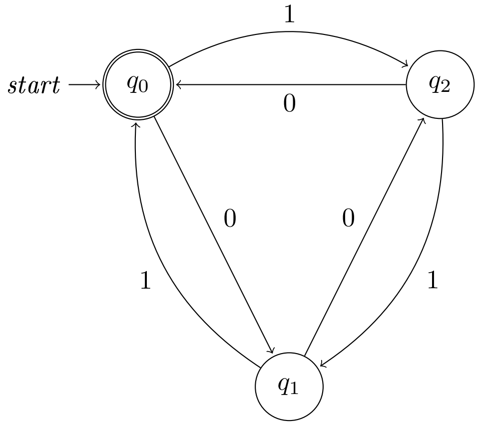
```
if x1 <= x2 then
    P
end;
```

21		no	*Is the function $x_0 := f(x_1, x_2)$ computed by P LOOP-computable?*
22	yes		*Is the function $x_0 := f'(x_1, x_2)$ computed by P' LOOP-computable?*
23	yes		*Are both f and f' μ-recursive?*

Part 5 $\boxed{\textit{Open2017}}$

((2 points))

Let $N = (Q, \Sigma, \delta, q_0, F)$ be a deterministic finite state machine with $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1\}$, $S = \{q_0\}$, $F = \{q_0\}$, and transition function δ as given below.



1. *Let X_i denote the regular expression for the language accepted by N when starting in state q_i.*

   Write down an equation system for $X_0, \ldots, X_2$.

2. *Give a regular expression r such that $L(r) = L(N)$ (you may apply Arden's Lemma to the result of 1).*

$$
\begin{aligned}
X_0 &= 0X_1 + 1X_2 + \varepsilon \\
X_1 &= 1X_0 + 0X_2 \\
X_2 &= 0X_0 + 1X_1 \\
r &= ((0 + 11)(01)^*(1 + 00) + 10)^* \\
\textit{alternatively}: & \\
r &= (01 + (00 + 1)(10)^*(0 + 11))^*
\end{aligned}
$$