Problems Solved:

31	32	33	34	35

Name:

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Problem 31. Let Σ be an alphabet and A be a set ($A \subseteq \Sigma^*$). Let also A be semi-decidable, but not decidable. Prove that the complement of A, i. e., $\bar{A} = \Sigma^* \setminus A$, is not decidable.

Problem 32. Let L be a finite language over an alphabet $\{0, 1\}$. Is the following problem (with input $\langle M \rangle$)

For a Turing maschine M it holds $L(M) \supseteq L$.

in general semi-decidable? Is it also in general decidable? Justify your answers.

Problem 33. Which of the following problems are decidable? Justify your answers. In each problem below, the input of the problem is the code $\langle M \rangle$ of a Turing machine M with input alphabet $\{0, 1\}$.

1. Is $L(M)$ empty?
2. Is $L(M)$ finite?
3. Is $L(M)$ regular?
4. Is $L(M) \subseteq \{0, 1\}^*$?
5. Is $L(M)$ not recursively enumerable?
6. Does M have an even number of states?

Problem 34. For a Turing machine M let $P(M)$ be the following property: *If M runs at least 1000 steps on a word w, then $w \in L(M)$.* Note that there is no statement about acceptance or non-acceptance if the machine runs less than 1000 steps.
In the following let M be a Turing machine that has the property $P(M)$.

1. Is there a Turing machine E with $P(E)$ such that $\varepsilon \in L(E)$.
2. Is there a Turing machine E with $P(E)$ such that $\varepsilon \notin L(E)$.
3. Is the property of $L(M)$ to contain the empty word, decidable?
4. Is $L(M)$ recursively enumerable?
5. Is the complement $\overline{L(M)}$ recursively enumerable?
6. Is $L(M)$ recursive?
7. Is $L(M)$ necessarily finite?
8. Is $L(M)$ necessarily infinite?

Problem 35. Show that the Acceptance Problem is reducible to the restricted Halting problem. First explain clearly which Turing machine you have to construct to prove this statement and then give a reasonably detailed description of this construction.