Problems Solved:

26	27	28	29	30

Name:

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Problem 26. Consider the following term rewriting system:

$$\begin{array}{ll} \text{(Rule 1)} & p(x, s(y)) \to p(s(s(x)), y) \\ \text{(Rule 2)} & p(x, 0) \to s(x) \end{array}$$

1. Show that

$$p(0, s(p(0, s(0)))) \xrightarrow{*} s(s(s(s(s(s(s(s(s(0)))))))))$$

   by a suitable reduction sequence. For each reduction step, underline the subterm that you reduce, and indicate the reduction rule and the matching substitution σ used explicitly.

2. Prove or disprove (an informal argument suffices)

$$p(0, p(0, p(0, p(0, s(0))))) \xrightarrow{*} \underbrace{s(s(s(s(s(s(s(s(s(s(s(s(s(s(s(s(}_{16\times s}0)))))))))))))))).$$

Problem 27. Consider the grammar $G = (N, \Sigma, P, S)$ where $N = \{S\}$, $\Sigma = \{a, b\}$, $P = \{S \to \varepsilon, S \to aSbS\}$.

(a) Is $aababb \in L(G)$?

(b) Is $aabab \in L(G)$?

(c) Does every element of $L(G)$ contain the same number of occurrences of a and b?

(d) Is $L(G)$ regular?

(e) Is $L(G)$ recursive?

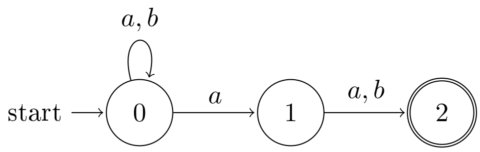
Justify your answers.

Problem 28.

(a) The NFSM

$$A = (Q, \Sigma, \delta, Q_0, F)$$

over the alphabet $\Sigma = \{a, b\}$ with the states $Q = \{0, 1, 2\}$, and the starting states $Q_0 = \{0\}$ and accepting states $F = \{2\}$ is given by the following picture.



Give a right-linear grammer $G = (N, \Sigma, P, S)$ with $L(G) = L(A)$ and give a derivation for the sentence *bab*.

(b) Now, let $A = (Q, \Sigma, \delta, Q_0, F)$ be an *arbitrary* NFSM. Give a right linear grammar $G = (N, \Sigma, P, S)$ with $L(G) = L(A)$.

Problem 29. Construct a NFSM recognizing $L(G)$ where $G = (\{A, B\}, \{a, b\}, P, A)$ with the production rules P given by

$$\begin{aligned} S &\to aS|bA|b, \\ A &\to aA|bS|a. \end{aligned}$$

Problem 30. According to Definition 32 of the lecture notes, there are no natural numbers in Lambda calculus. However, natural numbers can be encoded (known as Church encoding) as "Church numerals" (see below), i.e., as functions $\mathbf{n}$ that map any function f to its n-fold application $f^n = f \circ \ldots \circ f$. Note that we denote such a "natural number" representation via boldface symbols in order to emphasize that these are lambda terms. In other words, we define Church numerals as follows. By letting "application" bind stronger than "abstraction", we avoid writing parentheses where appropriate.

$$\begin{aligned} \mathbf{0} &= \lambda f.\lambda x.x \\ \mathbf{1} &= \lambda f.\lambda x.fx \\ \mathbf{2} &= \lambda f.\lambda x.f(fx) \\ \mathbf{3} &= \lambda f.\lambda x.f(f(fx)) \\ \mathbf{4} &= \lambda f.\lambda x.f(f(f(fx))) \\ &\vdots \\ \mathbf{n} &= \lambda f.\lambda x.\underbrace{f(\cdots(f}_{n\text{-fold}}\,x)\cdots) \end{aligned}$$

1. Define a lambda term add that represents addition of "Church numerals".
2. Show the intermediate steps of a reduction from ((add $\mathbf{2}$) $\mathbf{1}$) to $\mathbf{3}$.

Hint: a bit of literature research may help.