

**Problems Solved:**

11	12	13	14	15
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**Name:**

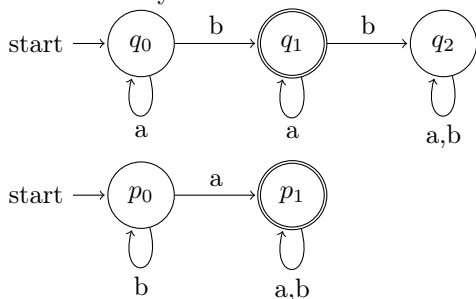
**Matrikel-Nr.:**

**Problem 11.** Let  $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$  and  $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$  be two DFSM over the alphabet  $\Sigma$ . Let  $L(M_1)$  and  $L(M_2)$  be the languages accepted by  $M_1$  and  $M_2$ , respectively.

Construct a DFSM  $M = (Q, \Sigma, \delta, q, F)$  whose language  $L(M)$  is the intersection of  $L(M_1)$  and  $L(M_2)$ . Write down  $Q$ ,  $\delta$ ,  $q$ , and  $F$  explicitly.

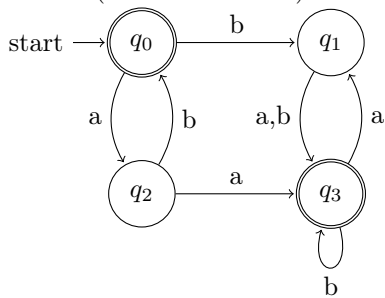
*Hint:*  $M$  simulates the parallel execution of  $M_1$  and  $M_2$ . For that to work,  $M$  “remembers” in its state the state  $M_1$  as well as the state of  $M_2$ . This can be achieved by defining  $Q = Q_1 \times Q_2$ .

Demonstrate your construction with the following DFSMs.



**Problem 12.** Show that the language  $L = \{a^m b^n \mid m, n \in \mathbb{N} \wedge m \geq 2n\}$  is not regular.

**Problem 13.** Let  $M_1$  be the DFSM with states  $\{q_1, q_2, q_3, q_4\}$  whose transition graph is given below. Determine a regular expression  $r$  such that  $L(r) = L(M_1)$ . Show the *derivation* of the the final result by the technique based on Arden’s Lemma (see lecture notes).



**Problem 14.** Let  $r$  be the following regular expression.

$$a \cdot a \cdot (b \cdot a)^* \cdot b \cdot b^*$$

Construct a nondeterministic finite state machine  $N$  such that  $L(N) = L(r)$ . Show the derivation of the result by following the technique presented in the proof of the theorem *Equivalence of Regular Expressions and Automata* (see lecture notes).

**Problem 15.** Write down explicitly a Turing machine  $M$  over  $\Sigma = \{0\}$  which computes the function  $d : \mathbb{N} \rightarrow \mathbb{N}$  given by  $d(n) = 2n$ .

Use unary representation: A number  $n$  is represented by the string  $0^n$  consisting of  $n$  copies of the symbol 0.