Gruppe	Hemmecke (10:15)	Hemmecke (11:00)	Popov
Name		Matrikel	SKZ

Klausur 1

Berechenbarkeit und Komplexität

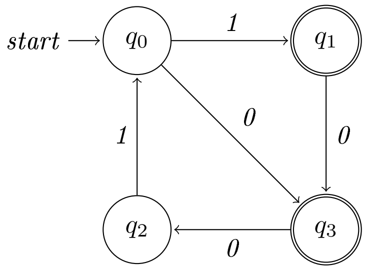
18. November 2016

Part 1 $\boxed{\textit{NFSM2016}}$

Let N be the nondeterministic finite state machine

$$(\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \nu, \{q_0\}, \{q_1, q_3\}),$$

whose transition function ν is given below.



1		no

Is $10010011001101 \in L(N)$*?*

A word $w \in L(N)$ with $|w| > 1$ ends either with 11 or 10, but never with 01.

2	yes	

Is $00110010 \in L(N)$*?*

Follow the states $q_0, q_3, q_2, q_0, q_1, q_3, q_2, q_0, q_3$.

3		no

Is $L(N)$ finite?

4	yes	

Does there exist a regular expression r such that $L(r) = \overline{L(N)} = \{0, 1\}^* \setminus L(N)$?

$L(N)$ is regular and so is its complement.

5	yes	

Is $\overline{L(N)}$ recursively enumerable?

$L(N)$ is regular. Hence, $\overline{L(N)}$ is regular, and thus also recursively enumerable.

6	yes	

Is there a deterministic finite state machine M with less than 2016 states such that $L(M) = L(N)$?

According to the subset construction, there must be a DFSM with at most $2^4 = 16$ states.

7	yes	

Is there an enumerator Turing machine G such that $Gen(G) = L(N)$?

8	yes	

Does there exists a deterministic finite state machine D such that $L(D) = L(N) \circ \overline{L(N)}$?

$L(N)$ and $\overline{L(N)}$ are both regular. Concatenation of two regular languages gives a regular language.

Part 2 $\boxed{\textit{Computable2016}}$

Let M be a Turing machine such that it accepts a word, if and only if it is a palindrome. A palindrome is a word that can be read the same way from either *direction, left-to-right or right-to-left. For example,* noon, civic, madam, *and* radar *are palindromes.*

9	yes		*Is $L(M)$ recursively enumerable?*
10	yes		*Is $L(M)$ recursive?*
11		no	*Is $L(M)$ finite?*

There can be arbitrarily large palindromes.

12		no	*Let L be a recursively enumerable language. Can it be concluded that $L(M) \cap L$ is recursive?*

Intersection of recursive and recursively enumerable languages is recursively enumerable but not necessarily recursive.

13		no	*Is every μ-recursive function also a primitive recursive function?*
14		no	*Does there exist a μ-recursive function that is not WHILE computable?*
15	yes		*Is every primitive recursive function also Turing-computable?*

Part 3 *Pumping2016*

Let

$$L_1 = \left\{ a^n b^m a^{n-m} \mid n, m \in \mathbb{N}, n > m, n < 2016 \right\} \subset \{a, b\}^*,$$
$$L_2 = \left\{ a^m b^n a^{n+m} \mid m, n \in \mathbb{N}, m > n > 1 \right\} \subset \{a, b\}^*.$$

16	yes		*Is there a deterministic finite state machine M such that $L(M) = L_1$?*

The language L_1 is finite and thus regular.

17		no	*Is there a deterministic finite state machine M' such that $L(M') = L_2$?*
18	yes		*Is there an enumerator Turing machine G such that $Gen(G) = L_2$?*
19	yes		*Is there a deterministic finite state machine D such that $L(D) = L_1 \cap L_2$?*

The language $L_1 \cap L_2$ is finite and thus regular.

20	yes		*Is there a language L such that $L \cup L_2$ is regular?*

Yes. Take as L the complement of L_2.

Part 4 *WhileLoop2016*

Let a function $f : \mathbb{N}^3 \to \mathbb{N}$ be defined by

$$f(x, y, z) := \begin{cases} y & \text{if } x = y, \\ z & \text{if } x < y, \\ 0 & \text{otherwise.} \end{cases}$$

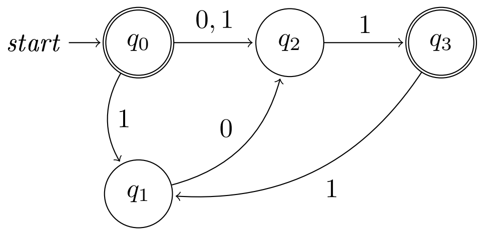
Let f' be defined like f, but with the exception that f' is undefined if one of the arguments is equal to 2016.

21	yes		*Is f a LOOP computable function?*
22		no	*Is f' a LOOP computable function?*
23	yes		*Is f' a WHILE computable function?*

Part 5 *Open2016*

((2 points))

Let $N = (Q, \Sigma, \delta, q_0, F)$ be a nondeterministic finite state machine with $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $S = \{q_0\}$, $F = \{q_0, q_3\}$, and transition function δ as given below.



1. *Let X_i denote the regular expression for the language accepted by N when starting in state q_i.*

   Write down an equation system for $X_0, \ldots, X_3$.

2. *Give a regular expression r such that $L(r) = L(N)$ (you may apply Arden's Lemma to the result of 1).*

$$
\begin{aligned}
X_0 &= 1X_1 + (0+1)X_2 + \varepsilon \\
X_1 &= 0X_2 \\
X_2 &= 1X_3 \\
X_3 &= 1X_1 + \varepsilon \\
r &= 1(011)^*01 + (0+1)(110)^*1 \\
&= ((0+1) + 10)(110)^*1 + \varepsilon
\end{aligned}
$$