Problems Solved:

26	27	28	29	30

Name:

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Problem 26. Consider the following term rewriting system:

$$a(x, s(y)) \to a(s(x), y) \quad (1)$$
$$a(x, 0) \to x \quad (2)$$
$$m(x, s(y)) \to a(m(x, y), x) \quad (3)$$
$$m(x, 0) \to 0 \quad (4)$$

Show that

$$m(s(s(0)), s(0)) \xrightarrow{*} s(s(0))$$

by a suitable reduction sequence. For each reduction step, underline the subterm that you reduce, and indicate the reduction rule and the matching substitution σ used explicitly.

Problem 27. Consider the grammar $G = (N, \Sigma, P, S)$ where $N = \{S\}$, $\Sigma = \{a, b\}$, $P = \{S \to \varepsilon, S \to aSbS\}$.

(a) Is $aababb \in L(G)$?

(b) Is $aabab \in L(G)$?

(c) Does every element of $L(G)$ contain the same number of occurrences of a and b?

(d) Is $L(G)$ regular?

(e) Is $L(G)$ recursive?

Justify your answers.

Problem 28. Define the following languages by context-free grammars over the alphabet $\Sigma = \{0, 1\}$.

(a) $L_1 = \{w \mid w \text{ contains at least two zeroes.}\}$

(b) $L_2 = \{w \mid w \text{ starts and ends with one and the same symbol.}\}$
Note that one-letter words are in L_2.

(c) $L_3 = \{w \mid w \text{ consists of an odd number of symbols and the symbol in the center of } w \text{ is a } 0.\}$

(d) $L_4 = L_2 \cap L_3$

Problem 29. Construct a DFSM recognizing $L(G)$ where $G = (\{A, B, C\}, \{a, b, c\}, P, A)$ with the production rules P given by

$$A \to aA|bA|cA|bB|cB,$$
$$B \to cA|cB|b|c.$$

Hint: Start by a constructing a NFSM N. Then turn N into a DFSM D such that $L(G) = L(N) = L(D)$.
"Construct" means to explain how you turn the grammar into a DFSM. Simply writing down a DFSM D with the required property, does not count as a solution unless you *prove* that $L(G) = L(D)$.

Problem 30. According to Definition 32 of the lecture notes, there are no natural numbers in Lambda calculus. However, natural numbers can be encoded (known as Church encoding) as "Church numerals" (see below), i.e., as functions $\mathbf{n}$ that map any function f to its n-fold application $f^n = f \circ \ldots \circ f$. Note that we denote such a "natural number" representation via boldface symbols in order to emphasize that these are lambda terms. In other words, we define Church numerals as follows. By letting "application" bind stronger than "abstraction", we avoid writing parentheses where appropriate.

$$\begin{aligned}
\mathbf{0} &= \lambda f.\lambda x.x \\
\mathbf{1} &= \lambda f.\lambda x.fx \\
\mathbf{2} &= \lambda f.\lambda x.f(fx) \\
\mathbf{3} &= \lambda f.\lambda x.f(f(fx)) \\
\mathbf{4} &= \lambda f.\lambda x.f(f(f(fx))) \\
&\vdots \\
\mathbf{n} &= \lambda f.\lambda x.\underbrace{f(\cdots(f\,x)\cdots)}_{n\text{-fold}}
\end{aligned}$$

1. Define a lambda term add that represents addition of "Church numerals".
2. Show the intermediate steps of a reduction from ((add $\mathbf{2}$) $\mathbf{1}$) to $\mathbf{3}$.