Problems Solved:

1	2	3	4	5

Name:

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Problem 1. A function f is given by $f(0) = 1$ and $f(n+1) = 2f(n)$ for $n \geq 0$. Show by induction that $f(n) = 2^n$ for $n \geq 0$.

Problem 2. Let $L \subseteq \Sigma^*$ be a language over the alphabet $\Sigma = \{a, b, c, d\}$ such that a word w is in L if and only if it is either a or b or of the form $w = ducvd$ where u and v are words of L. For example, *dacad*, *ddacbdcad*, *dddbcbdcdbcbddcad* are words in L. Show by induction that every word of L contains an even number of the letter d.
Note that a *language* is just a set of words and a *word* is simply a finite sequence of letters from the alphabet.

Problem 3. Show that

$$\forall x \in \mathbb{R}\ \forall y \in \mathbb{R} : \quad x^2 + y^4 = 10 \implies x \leq 4.$$

Hint: (indirect proof) In order to show $A \implies B$ one may assume A and also the opposite of B, i.e., $\neg B$ and from these two assumptions derive a contradiction.

Problem 4. Construct a deterministic finite state machine M over the alphabet $\{a, b, c\}$ such that it accepts the language $L(M) = \{abc\}$.

(a) Draw the graph.

(b) Provide the components of the defining quintuple explicitly.

(c) What has to be changed in order for the machine to accept all finite repetitions of the string *abc*? (The empty word shall not be accepted.)

Problem 5. Construct a nondeterministic finite state machine for:

1. the language L_1 of all strings over $\{0, 1\}$ that contain 100 as a substring.
2. the language L_2 of all strings over $\{0, 1\}$ that contain the letters 1, 0, 0 in exactly that order. (Note that before, in between and after these three letters any number of other letters may occur).

Your two machines must not use more than 4 states. Moreover, they should only differ in their transition functions. Draw their transition graphs.