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A Parallel, In-Place, Rectangular Matrix Transpose Algorithm

Computational Complexity Analysis

Table of Contents

1. Introduction
2. Revision of TRIP
3. Analysis of Computational Complexity
    a. Work
    b. Span
    c. Parallelism
    d. Generalizations

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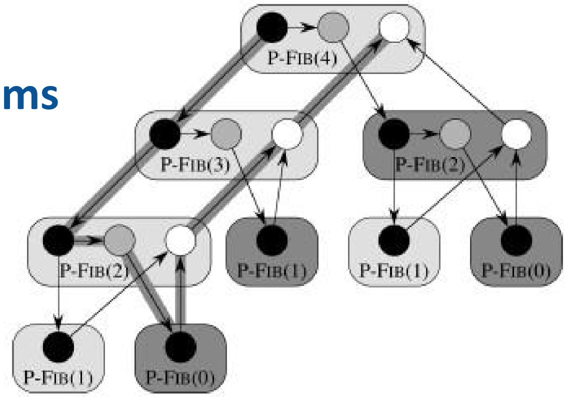
Introduction

Introduction

Computational Complexity of Parallel Algorithms



Introduction to Algorithms Third Edition, p777ff

Work W_1

"execution time on one processor"

i.e.: **all vertices** of computation dag

approximation: # of nodes of computation dag

Span W_∞

"execution time on infinitely many processors"

i.e. length of **critical path** of computation dag

approximation: # of nodes on critical path

Parallelism $\frac{W_1}{W_\infty}$

- average amount of work per step along critical path
- maximum possible speedup
- limit on possibility of attaining perfect speedup

Revision of TRIP

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TRIP:
If matrix is rectangular ***TRIP*** transposes sub-matrices, then combines the result with ***merge*** or ***split***

merge:
first rotates the middle part of the array, then recursively merges the left and right parts of the array

split:
first recursively splits the left and right parts of the array, then rotates the middle part of the array

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Analysis of Computational Complexity

Restriction to Powers of Two

"power condition"

Matrix dimensions M x N and N x M

$$\left(\exists k \in \mathbb{N} : N = 2^k\right) \wedge \left(\exists l \in \mathbb{N} : M = 2^l N\right)$$

recursive calls are symmetric

TRIP's recursive call are either all `merge` or all `split`

Work

1. Example: Basic Algorithms
2. TRIP Result & Proof Sketch

Work Example

Work of Base Algorithms

```
cilk reverse(A, m0, m1, l) {
  if (l > 1) {
    lm = l/2;
    spawn reverse(A, m0, m1, lm);
    spawn reverse(A, m0 + lm, m1 - lm, l - lm);
  } else {
    swap(A, m0, m1 - 1);
  }
}
```

Lemma 1 (Work of reverse) *If length $n = m_1 - m_0$ of array A is even, then*

$$W_1^{reverse}(n) = n - 1$$

Proof. initially $l = n/2$, base case $l = 1$

binary tree has $n/2 - 1$ nodes

1 swap per leaf: $n/2$ swaps

$$W_1^{\text{reverse}}(n) = \frac{n}{2} - 1 + \frac{n}{2} = n - 1$$

□

```
cilk rol(A, n, k) {
  spawn reverse(a, 0, k);
  spawn reverse(a, k, n);
  sync;
  spawn reverse(a, 0, n);
}
```

Lemma 2 (Work of rol) *If length n of array A is even, then the work of rol$(n,\ n/2)$ is*

$$W_1^{rol}(n,\ n/2) = 2n - 3$$

Proof. algorithm not recursive

half array rotations: work $n/2 - 1$ each
full array rotation: work $n - 1$

$$W_1^{\text{rol}}(n,\ n/2) = 2\left(\frac{n}{2} - 1\right) + (n - 1) = 2n - 3$$

□

Result

Work of TRIP

Show that under power condition, for M x N matrix

$$W_1^{\mathrm{TRIP}}(M,\ N) = \Theta\left(MN\left(1 + \log\frac{M}{N}\ \log\ N\right)\right)$$

and in general:

$$W_1^{\mathrm{TRIP}}(M,\ N) = \Theta\left(MN\left(1 + \log\frac{\max(M,\ N)}{\min(M,\ N)}\ \log\ \min(M,\ N)\right)\right)$$

METHOD

don't count vertices in computation dag

count **inner nodes in recursion tree** and **swaps**

- function calls (nodes in recursive call trees)
- memory accesses (swaps)

Work of TRIP

Proof Sketch

Work of TRIP

$$W_1^{\mathrm{TRIP}}(M,\ N) = \underbrace{\frac{M}{N} - 1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0 \le i < \lg \frac{M}{N}} 2^i W_1^{\mathrm{merge}}(p_i,\ q_i,\ N)}_{\text{combine effort}} + \underbrace{\frac{M}{N} W_1^{\mathrm{square}}(N)}_{\text{work in leaves of transpose tree (square\_transpose)}}$$

Work of merge

$$W_1^{\mathrm{merge}}(p_i,\ q_i,\ N) = \underbrace{N - 1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0 \le j < \lg N} 2^j W_1^{\mathrm{rol}}\left(\frac{n_j}{2}p_i + \frac{n_j}{2}q_i,\ \frac{n_j}{2}p_i\right)}_{\text{work within inner nodes of the merge tree}}$$

Work of rol

$$W_1^{\mathrm{rol}}\left(\frac{n_j}{2}p_i + \frac{n_j}{2}q_i,\ \frac{n_j}{2}p_i\right) = NM2^{-i-j} - 3$$

Work of TRIP

wide matrices

TRIP recursion is analogous for tall and wide matrices

only difference:

- in `merge` `rol` is called before the recursive merge call
- in `splitt` `rol` is called after the recursive split call

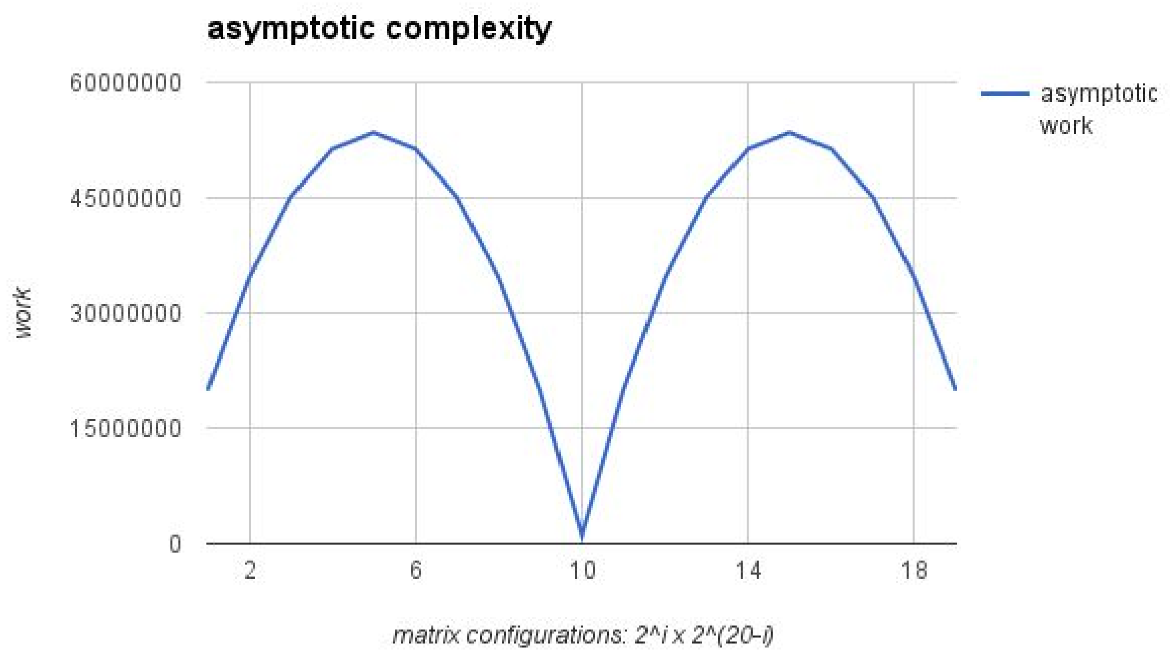
This difference does not cause a change in the amount of work of `TRIP`.

Corollary 1 (Work of TRIP). *Let $A_{M,N}$ be a tall, wide or square matrix that satisfies the power condition. Then*

$$W_1^{TRIP}(M,\ N) = \Theta\left(MN\left(1 + \log\frac{\max(M,\ N)}{\min(M,\ N)}\ \log\ \min(M,\ N)\right)\right)$$

Visualization

Work as function of Matrix Dimensions



Span

1. Example: Basic Algorithms
2. TRIP Result

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Span Example

Span of Base Algorithms

```
cilk reverse(A, m0, m1, l) {
  if (l > 1) {
    lm = l/2;
    spawn reverse(A, m0, m1, lm);
    spawn reverse(A, m0 + lm, m1 − lm, l − lm);
  } else {
    swap(A, m0, m1 − 1);
  }
}
```

Lemma 3 (Span of **reverse**). *If length n of array A is a power of two, then*

$$W_\infty^{reverse}(n) = \log\ \frac{n}{2} + 1$$

Proof. initially $l = n/2$, base case $l = 1$

binary tree has log $n/2$ levels

1 swap in leaf adds 1 to span

$$W_\infty^{\texttt{reverse}}(n) = \log\ \frac{n}{2} + 1$$

□

```
cilk rol(A, n, k) {
  spawn reverse(a, 0, k);
  spawn reverse(a, k, n);
  sync;
  spawn reverse(a, 0, n);
}
```

Lemma 4 (Span of `rol`). *If length n of array A is a power of two, then the span of rol$(n,\ n/2)$ is*

$$W_\infty^{rol}(n) = \log\ 2^{-1}n + \log\ 2^{-2}n + 3$$

Proof. algorithm not recursive

`rol` consists of three reversals of lengths $n/2$, $n/2$ and n
first two are in parallel
Lemma 3
adding span 1 for high-level function calls (cf. inner nodes)

$$W_\infty^{\mathrm{rol}}(n,\ n/2) = \log\ \frac{n}{2} + \log\ \frac{n}{4} + 3 = \log\ 2^{-1}n + \log\ 2^{-2}n + 3$$

□

Span of TRIP

Result

Calculate span of tall matrix transpose

count **levels** and **swaps** on critical path, that includes

span of

- creating the divide tree
- combining the nodes via `merge`/`split` (itself recursive procedures)
- square-transposing in the leaf nodes

Theorem 1 (Span of TRIP for tall matrices)**.** *Let $A_{M,N}$ be a* ***tall*** *matrix that satisfies the power condition.*

Then

$$W_{\infty}^{TRIP}(M,\ N) = \log\ \frac{M}{N} + \log\ \frac{M}{N}\ \log\ N\ \left(3\ \log\ N + \log\ \frac{M}{N}\right) + \log\ N + 1$$

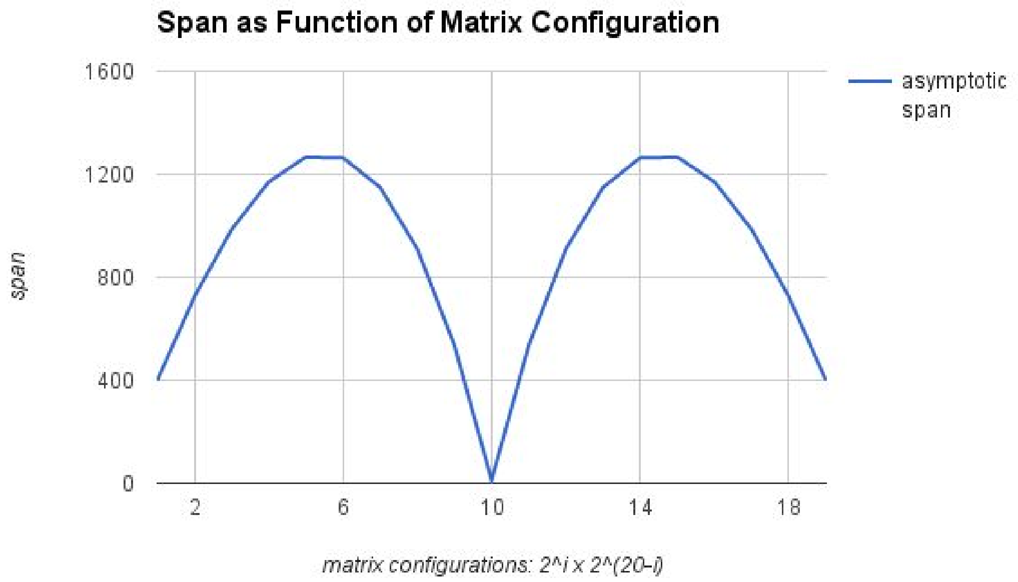
in general

$$W_{\infty}^{\mathrm{TRIP}}(M,\ N) = \log\ \frac{m}{n} + \log\ \frac{m}{n}\ \log\ n\ \left(3\ \log\ n + \log\ \frac{m}{n}\right) + \log\ n + 1$$

where $m = \max(M,\ N)$ and $n = \min(M,\ N)$

Visualization

Span as function of Matrix Dimensions



Parallelism

Result

Rectangular Matrices

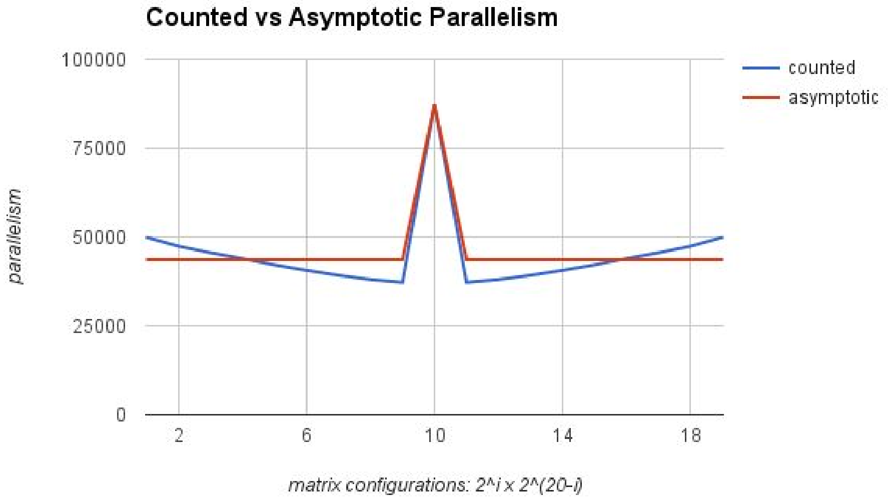
$$\Theta\left(\frac{MN}{\log\ M/N+\log\ N}\right)$$

Square Matrices

$$\Theta\left(\frac{N^2}{\log\ N}\right)$$

calculation:

- divide work by span
- case distinction rectangular / square
- simplification using Landau symbols



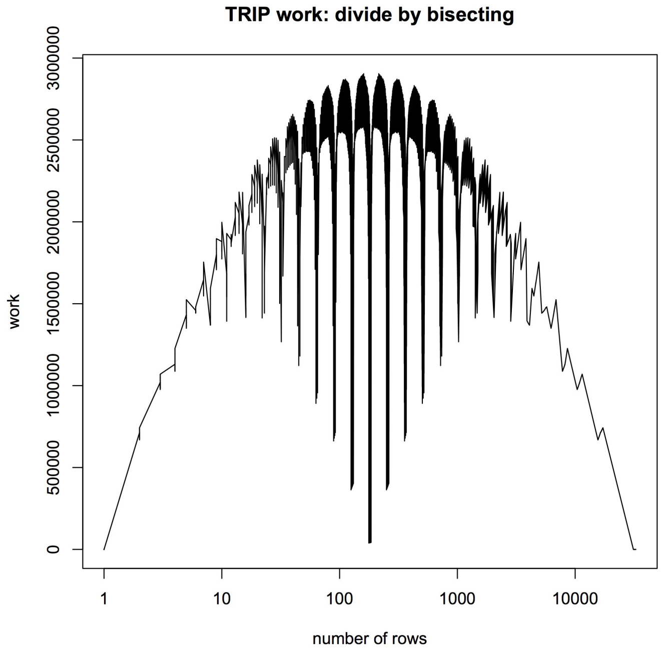
Generalizations

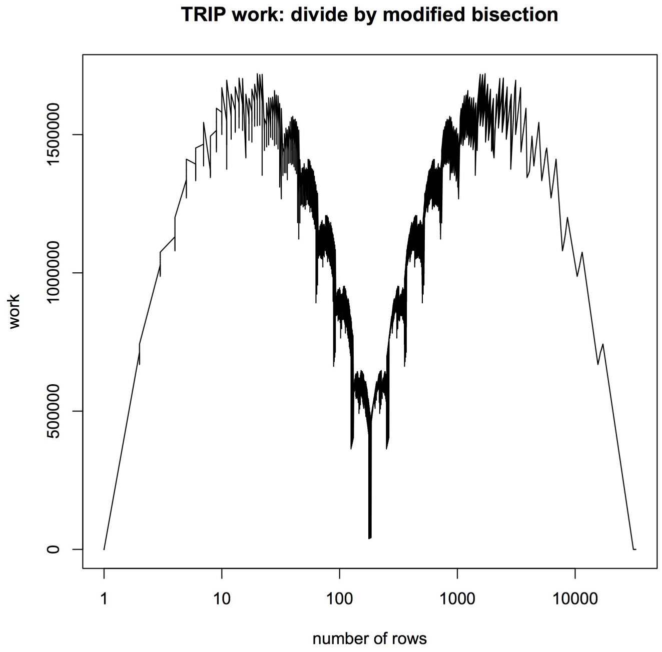
power condition unsatisfied

Example: 7 x 5 Matrix

Generalization

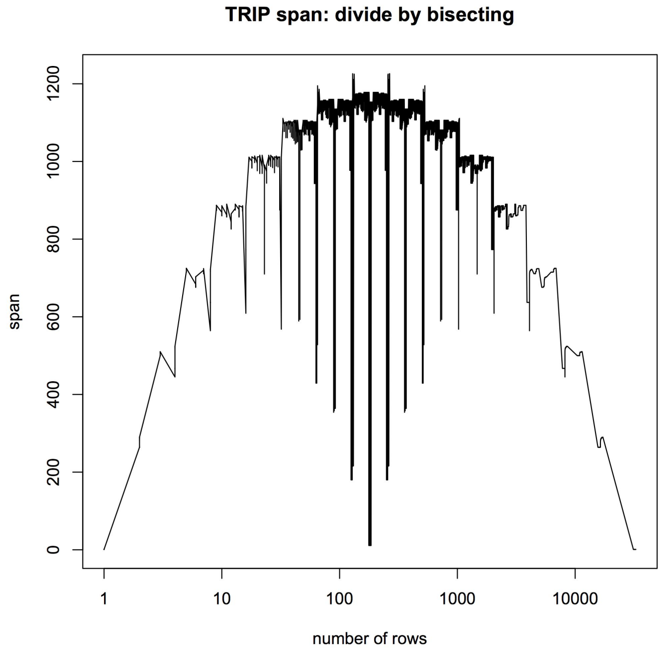
Power Condition not Satisfied

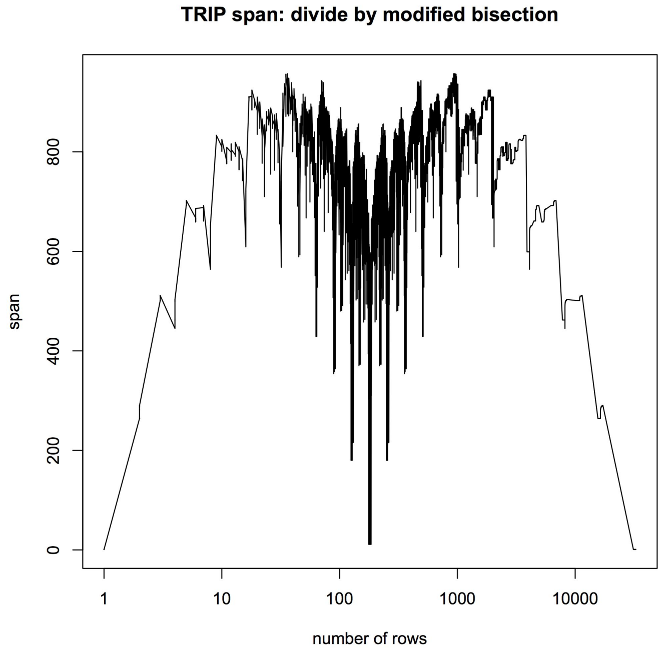




Generalization

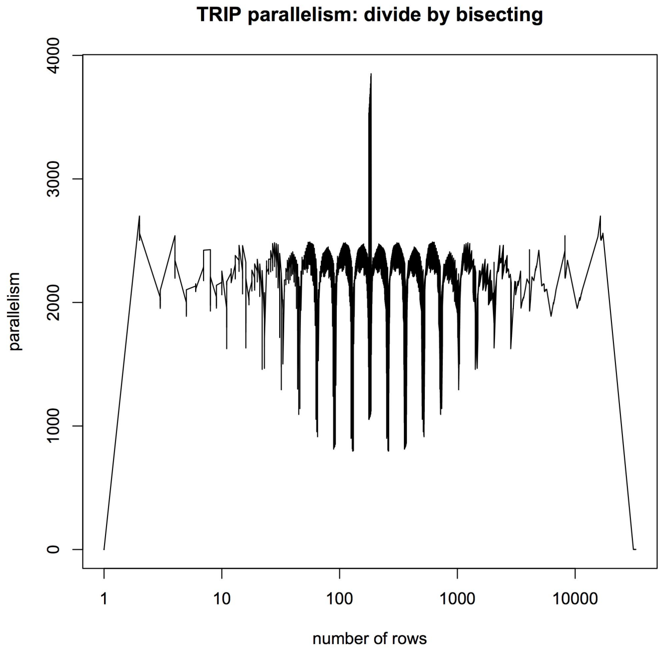
Power Condition not Satisfied

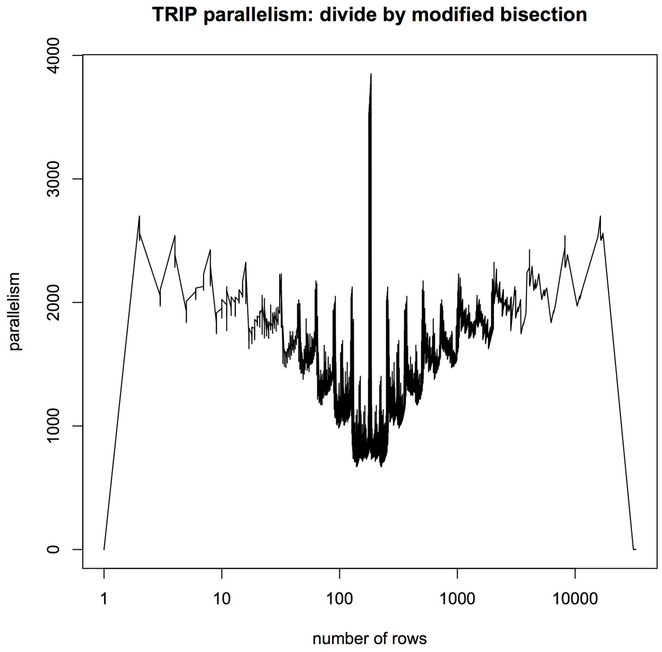




Generalization

Power Condition not Satisfied





Thank you

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Revision of TRIP

TRIP Algorithm

If matrix is rectangular ***TRIP*** transposes sub-matrices, then combines the result with ***merge*** or ***split***

$$
\mathrm{TRIP}(A,\ m,\ n) = \begin{cases}
\begin{array}{l}\mathrm{TRIP}(A(0:\lfloor \frac{m}{2} \rfloor,\ 0:n),\ \lfloor \frac{m}{2} \rfloor,\ n) \parallel \\ \mathrm{TRIP}(A(\lfloor \frac{m}{2} \rfloor : m,\ 0:n),\ \lceil \frac{m}{2} \rceil,\ n); \\ \mathrm{merge}(\overline{A},\ \lfloor \frac{m}{2} \rfloor,\ \lceil \frac{m}{2} \rceil,\ n)\end{array} & \text{if } m > n \\
\\
\begin{array}{l}\mathrm{TRIP}(A(0:m,\ 0:\lfloor \frac{n}{2} \rfloor),\ m,\ \lfloor \frac{n}{2} \rfloor) \parallel \\ \mathrm{TRIP}(A(0:m,\ \lfloor \frac{n}{2} \rfloor : n),\ m,\ \lceil \frac{n}{2} \rceil); \\ \mathrm{split}(\overline{A},\ \lfloor \frac{n}{2} \rfloor,\ \lceil \frac{n}{2} \rceil,\ m)\end{array} & \text{if } m < n \\
\\
\mathrm{square\_transpose}(A,\ n) & \text{if } m = n
\end{cases}
$$

`merge` Algorithm

merge combines the transposes of sub-matrices of *tall* matrices

merge first rotates the middle part of the array, then recursively merges the left and right parts of the array

$$\operatorname{merge}(\overline{A},\ p,\ q,\ n) = \begin{cases} \operatorname{rol}(\overline{A}(\lfloor \frac{n}{2} \rfloor p : np + \lfloor \frac{n}{2} \rfloor q),\ \lceil \frac{n}{2} \rceil p); & \\ \operatorname{merge}(\overline{A}(0 : \lfloor \frac{n}{2} \rfloor (p+q)),\ p,\ q,\ \lfloor \frac{n}{2} \rfloor) \parallel & \text{if } n > 1 \\ \operatorname{merge}(\overline{A}(\lfloor \frac{n}{2} \rfloor (p+q) : n(p+q)),\ p,\ q,\ \lceil \frac{n}{2} \rceil) & \\ & \\ \overline{A} & \text{if } n = 1 \end{cases}$$

rol(arr, k) … left rotation (circular shift) of array ***arr*** by ***k*** elements

`split` Algorithm

split combines the transposes of sub-matrices of *wide* matrices

split first recursively splits the left and right parts of the array, then rotates the middle part of the array

$$\operatorname{split}(\overline{A},\ p,\ q,\ m) = \begin{cases} \operatorname{split}(\overline{A}(0 : \lfloor \frac{m}{2} \rfloor (p+q)),\ p,\ q,\ \lfloor \frac{m}{2} \rfloor) \parallel \\ \operatorname{split}(\overline{A}(\lfloor \frac{m}{2} \rfloor (p+q) : m(p+q)),\ p,\ q,\ \lceil \frac{m}{2} \rceil); & \text{if } \ m > 1 \\ \operatorname{rol}(\overline{A}(\lfloor \frac{m}{2} \rfloor p : mp + \lfloor \frac{m}{2} \rfloor q),\ \lfloor \frac{m}{2} \rfloor q) & \\ \\ \overline{A} & \text{if } \ m = 1 \end{cases}$$

split and ***merge*** are inverse to each other

Work Proof

Work of TRIP

Overview

Calculate work of tall matrix transpose

- spanning the divide tree
- combining the nodes via `merge`/`split` (itself recursive procedures)
- square-transposing in the leaf nodes

Theorem 1 (Work of `TRIP` for tall matrices). *Let $A_{M,N}$ be a **tall** matrix that satisfies the power-condition.*

Then

$$W_1^{\mathit{TRIP}}(M,\ N) = \Theta\left(MN\left(1 + \log\frac{M}{N}\ \log\ N\right)\right)$$

Work of TRIP

Proof - TRIP Tree

Spanning Divide Tree

recursion parameter of transpose is m, $m_0 = M$

$$\forall_{0 \le i < \lg \frac{M}{N}} m_{i+1} = m_i/2$$

base case is $m = N$ (square sub-matrix)

- $\lg \frac{M}{N}$ levels
- $\frac{M}{N}$ leafs
- $\frac{M}{N} - 1$ inner nodes

Combining Nodes via `merge`

number of inner nodes at level i is 2^i

at level i $\texttt{merge}(m_i/2,\ m_i/2,\ N)$ is called

$m_0 = M \Rightarrow m_i/2 = M2^{-(i+1)}$

$$W_1^{\text{TRIP}}(M,\ N) = \underbrace{\frac{M}{N} - 1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0 \le i < \lg \frac{M}{N}} 2^i \boxed{W_1^{\text{merge}}(p_i,\ q_i,\ N)}}_{\text{combine effort}} + \underbrace{\frac{M}{N} W_1^{\text{square}}(N)}_{\text{work in leaves of transpose tree (square\_transpose)}}$$

Work of TRIP

Proof - Merge Tree

Combining via `merge`, rotate sub-arrays

$$W_1^{\text{TRIP}}(M,\ N) = \underbrace{\frac{M}{N} - 1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0 \le i < \lg \frac{M}{N}} 2^i \boxed{W_1^{\text{merge}}(p_i,\ q_i,\ N)}}_{\text{combine effort}} + \underbrace{\frac{M}{N} W_1^{\text{square}}(N)}_{\text{work in leaves of transpose tree (square\_transpose)}}$$

recursion parameter n with $n_0 = N$

base case $n_j = 1$ for some j

$n_{j+1} = n_j/2$

$\Rightarrow n_j = N2^{-j}$ for $0 \le j < \log\ N$.

$$W_1^{\text{merge}}(p_i,\ q_i,\ N) = \underbrace{N-1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0 \le j < \lg\ N} 2^j W_1^{\text{rol}}\left(\frac{n_j}{2}p_i + \frac{n_j}{2}q_i,\ \frac{n_j}{2}p_i\right)}_{\text{work within inner nodes of the merge tree}}$$

- `merge` call has $\log\ N$ levels
- $N-1$ inner nodes
- 2^j inner nodes at level j of merge tree
- 1 `rol` call per inner node

$n_j = N2^{-j}$ for $0 \le j < \log\ N$

$p_i = q_i = m_i/2 = M2^{-(i+1)}$

Lemma 2

$$W_1^{\text{rol}}\left(\frac{n_j}{2}p_i + \frac{n_j}{2}q_i,\ \frac{n_j}{2}p_i\right) = W_1^{\text{rol}}\left(NM2^{-i-j-1},\ \frac{1}{2}NM2^{-i-j-1}\right)$$
$$= NM2^{-i-j} - 3$$

Work of TRIP

Proof - Merge Tree

$$W_1^{\text{TRIP}}(M,\ N) = \underbrace{\frac{M}{N} - 1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0\le i<\lg\frac{M}{N}} 2^i \boxed{W_1^{\text{merge}}(p_i,\ q_i,\ N)}}_{\text{combine effort}} + \underbrace{\frac{M}{N} W_1^{\text{square}}(N)}_{\text{work in leaves of transpose tree (square\_transpose)}}$$

Integrate `rol` result into `merge` work

$$W_1^{\text{merge}}(p_i,\ q_i,\ N) = \underbrace{N-1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0\le j<\lg N} 2^j W_1^{\text{rol}}\left(\frac{n_j}{2}p_i + \frac{n_j}{2}q_i,\ \frac{n_j}{2}p_i\right)}_{\text{work within inner nodes of the merge tree}}$$

$$W_1^{\text{rol}}\left(\frac{n_j}{2}p_i + \frac{n_j}{2}q_i,\ \frac{n_j}{2}p_i\right) = NM2^{-i-j} - 3$$

$$\begin{aligned}
W_1^{\text{merge}}(p_i,\ q_i,\ N) &= N-1+\sum_{0\le j<\log N} 2^j W_1\left(\text{rol}\left(\frac{n_j}{2}p_i + \frac{n_j}{2}q_i,\ \frac{n_j}{2}p_i\right)\right)\\
&= N-1+\sum_{0\le j<\log N} 2^j\left(NM2^{-i-j}-3\right)\\
&= N-1+\log(N)NM2^{-i}-3\sum_{0\le j<\log N} 2^j\\
&= N-1+\log(N)NM2^{-i}-3(N-1)\\
&= \log(N)NM2^{-i}-2(N-1)
\end{aligned}$$

Work of TRIP

Proof - TRIP Tree

$$W_1^{\text{TRIP}}(M,\ N) = \underbrace{\frac{M}{N} - 1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0 \le i < \lg \frac{M}{N}} 2^i W_1^{\text{merge}}(p_i,\ q_i,\ N)}_{\text{combine effort}}$$
$$+ \underbrace{\frac{M}{N} \boxed{W_1^{\text{square}}(N)}}_{\text{work in leaves of transpose tree (square\_transpose)}}$$

Integrate merge result into TRIP work

$$W_1^{\text{merge}}(p_i,\ q_i,\ N) = \log\ (N) N M 2^{-i} - 2(N-1)$$

$$W_1^{\text{TRIP}}(M,\ N) = (-2N+3)\left(\frac{M}{N} - 1\right) + \left(MN\ \lg\ \frac{M}{N}\ \lg\ N\right) + \frac{M}{N} W_1^{\text{square}}(N)$$

$$\begin{aligned}
W_1^{\text{TRIP}}(M,\ N) &= \left(\frac{M}{N} - 1\right) + \sum_{0 \le i < \lg \frac{M}{N}} 2^i W_1^{\text{merge}}(p_i,\ q_i,\ N) + \frac{M}{N} W_1^{\text{square}}(N) \qquad (1)\\
&= \left(\frac{M}{N} - 1\right) + \sum_{0 \le i < \lg \frac{M}{N}} 2^i\ \left(\lg\ (N) N M 2^{-i} - 2(N-1)\right) + \frac{M}{N} W_1^{\text{square}}(N)\\
&= \left(\frac{M}{N} - 1\right) + \left(\lg\ \frac{M}{N}\ \lg\ (N) N M - 2(N-1)\left(\frac{M}{N} - 1\right)\right) + \frac{M}{N} W_1^{\text{square}}(N)\\
&= (-2N+3)\left(\frac{M}{N} - 1\right) + \left(MN\ \lg\ \frac{M}{N}\ \lg\ N\right) + \frac{M}{N} W_1^{\text{square}}(N)
\end{aligned}$$

Work of TRIP

Proof - Square Transpose

$$W_1^{\mathrm{TRIP}}(M, N) = \underbrace{\frac{M}{N} - 1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0 \le i < \lg \frac{M}{N}} 2^i W_1^{\mathrm{merge}}(p_i, q_i, N)}_{\text{combine effort}} + \underbrace{\frac{M}{N} \boxed{W_1^{\mathrm{square}}(N)}}_{\text{work in leaves of transpose tree (square\_transpose)}}$$

Recap

```
cilk square_transpose(A, i0, i1, j0, j1) {
  if (i1 − i0 > 1) {
    im = (i0 + i1)/2;
    jm = (j0 + j1)/2;
    spawn square_transpose(A, i0, im, j0, jm);
    spawn square_transpose(A, i0, im, jm, j1);
    spawn square_transpose(A, im, i1, jm, j1);
    if (i1 ≤ j0)
      spawn square_transpose(A, im, i1, j0, jm);
  } else {
    for (j = j0; j < j1; j++) {
      swap(A[j, i0], A[i0, j]);
    }
  }
}
```

Work of TRIP

Proof - Square Transpose

$$W_1^{\text{TRIP}}(M,\ N) = \underbrace{\frac{M}{N} - 1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0\le i<\lg\ \frac{M}{N}} 2^i W_1^{\text{merge}}(p_i,\ q_i,\ N)}_{\text{combine effort}} + \underbrace{\frac{M}{N} \boxed{W_1^{\text{square}}(N)}}_{\text{work in leaves of transpose tree (square\_transpose)}}$$

tree covers upper right matrix: $N(N+1)/2$ leaves

Lower Bound on # of inner nodes

purely ternary tree

tree would have $\lceil \log_3(N(N+1)/2) \rceil$ levels

number of inner nodes:

$$\sum_{i=0}^{\lceil \log_3(N(N+1)/2)-1 \rceil} 3^i$$

Since

$$\sum_{k=0}^{m} 3^i = \frac{1}{2}\left(3^{m+1} - 1\right)$$

Number of inner nodes is about

$$\sum_{i=0}^{\log_3(N(N+1)/2)-1} 3^i = \frac{1}{2}\left(3^{\log_3(N(N+1)/2)} - 1\right) = \frac{N(N+1)}{4} - \frac{1}{2} = \Theta(N^2)$$

Upper Bound on # of inner nodes

purely quaternary tree

tree would have $\lceil \log_4(N(N+1)/2) \rceil$ levels

number of inner nodes:

$$\sum_{i=0}^{\lceil \log_4(N(N+1)/2)-1 \rceil} 4^i$$

Since

$$\sum_{k=0}^{m} 4^i = \frac{1}{3}\left(4^{m+1} - 1\right)$$

Number of inner nodes is about

$$\sum_{i=0}^{\log_4(N(N+1)/2)-1} 4^i = \frac{1}{3}\left(4^{\log_4(N(N+1)/2)} - 1\right) = \frac{N(N+1)}{6} - \frac{1}{3} = \Theta(N^2)$$

Work of TRIP

Proof - TRIP Tree

$$W_1^{\text{TRIP}}(M,\ N) = \underbrace{\frac{M}{N} - 1}_{\#\text{ of inner nodes}} + \underbrace{\sum_{0 \le i < \lg\ \frac{M}{N}} 2^i W_1^{\text{merge}}(p_i,\ q_i,\ N)}_{\text{combine effort}} + \underbrace{\frac{M}{N} W_1^{\text{square}}(N)}_{\text{work in leaves of transpose tree (square\_transpose)}}$$

Integrate `square` `transpose` result into TRIP work

work of square transpose (including swapping)

$$W_1^{\text{square}}(N) = \Theta(N^2) + N(N-1)/2 = \Theta(N^2)$$

$$\begin{aligned} W_1^{\text{TRIP}}(M,\ N) &= (-2N+3)\left(\frac{M}{N} - 1\right) + \left(MN\ \lg\ \frac{M}{N}\ \lg\ N\right) + \frac{M}{N} W_1^{\text{square}}(N) \\ &= (-2N+3)\left(\frac{M}{N} - 1\right) + \left(MN\ \lg\ \frac{M}{N}\ \lg\ N\right) + \frac{M}{N}\Theta(N^2) \end{aligned}$$

Which simplifies to

$$\begin{aligned} W_1^{\text{TRIP}}(M,\ N) &= \Theta\left(M + MN\ \log\ \frac{M}{N}\ \log\ N + MN\right) \\ &= \Theta\left(MN\left(1 + \log\ \frac{M}{N}\ \log\ N\right)\right) \end{aligned}$$

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Conclusions

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Novel Algorithm ***TRIP*** transposes rectangular matrices

- correctly
- in-place
- in highly parallel manner

Roadmap

1. Work
2. Span
3. Parallelism