Gruppe	Hemmecke (10:15)	Hemmecke (11:00)	Popov
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Klausur 1

Berechenbarkeit und Komplexität

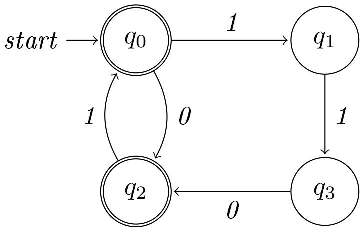
20. November 2015

Part 1 *NFSM2015*

Let N be the nondeterministic finite state machine

$$(\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \nu, \{q_0\}, \{q_0, q_2\}),$$

whose transition function ν is given below.



1		no

Is $1101100100100 \in L(N)$?

A word $w \in L(N)$ with $|w| > 1$ never ends with 00.

2		no

Is $L(N)$ finite?

3	yes	

Let N' be the NFSM that is constructed from N by solely reversing the arrow $q_2 \to q_0$ (the one with the letter 1) in the diagram above. Is $L(N')$ finite?

$L(N') = \{\epsilon, 0, 1, 110\}$.

4	yes	

Does there exist a regular expression r such that $L(r) = \overline{L(N)} = \{0, 1\}^* \setminus L(N)$?

$L(N)$ is regular and so is its complement.

5	yes	

Let R be a regular language and define $L = \{\, 01^n w \mid n \in \mathbb{N}, w \in R \,\}$. Is L a regular language?

6	yes	

Is there an enumerator Turing machine G such that $Gen(G) = L(N)$?

7	yes	

Is there a deterministic finite state machine M with less than 24 states such that $L(M) = L(N)$?

According to the subset construction, there must be a DFSM with at most $2^4 = 16$ states.

8	yes	

Is $L(N)^*$ recursively enumerable?

$L(N)$ is regular. Hence, $L(N)^*$ is regular, and thus also recursively enumerable.

Part 2 *Computable2015*

Let M be a Turing machine such that whenever M accepts a word, it does so in no more than 2015 steps.

9	yes		*Is $L(M)$ recursively enumerable?*
10	yes		*Is $L(M)$ recursive?*

Start M with input w and execute 2015 steps. If w has been accepted then $w \in L(M)$, otherwise $w \notin L(M)$. Therefore, $L(M)$ and $\overline{L(M)}$ are both recursively enumerable.

11		no	*Let L be a recursively enumerable language. Can it be concluded that $L(M) \cap L$ is recursive?*

Intersection of recursive and recursively enumerable languages is recursively enumerable but not necessarily recursive.

12	yes		*Is every primitive recursive function also a μ-recursive function?*
13		no	*Does there exist a μ-recursive function that is not WHILE computable?*
14	yes		*Let f be a primitive recursive function and $g : \{\sharp\}^* \to \{\sharp\}^*$ be defined by $g(\sharp^n) = \sharp^{f(n)}$ for all $n \in \mathbb{N}$. Is g Turing-computable?*

Part 3 *Pumping2015*

Let

$$L_1 = \left\{ a^{(3n+1)}b^n \,\middle|\, n \in \mathbb{N}, n < 2015 \right\} \subset \{a,b\}^*,$$
$$L_2 = \{ a^m b^n \mid m,n \in \mathbb{N}, m > n > 1 \} \subset \{a,b\}^*.$$

15	yes		*Is there a deterministic finite state machine M such that $L(M) = L_1$?*

The language L_1 is finite and thus regular.

16		no	*Is there a deterministic finite state machine M' such that $L(M') = L_2$?*
17	yes		*Is there an enumerator Turing machine G such that $Gen(G) = L_1$?*
18	yes		*Is there a deterministic finite state machine D such that $L(D) = L_1 \cap L_2$?*

The language $L_1 \cap L_2$ is finite and thus regular.

19	yes		*Is there a language L such that $L \cup L_2$ is regular?*
20	yes		*Are there two languages X_1 and X_2 that are not regular, but for which $X_1 \cup X_2$ is regular?*

Yes. Take $X_1 = \{ a^n b^n \mid n \in \mathbb{N} \}$ and $X_2 = \{ a^n b^m \mid n,m \in \mathbb{N}, n \neq m \}$.

Part 4 *WhileLoop2015*

Let T_1 and T_2 be two Turing machines. Assume that T_1 and T_2 compute partial functions $t_1, t_2 : \mathbb{N} \to \mathbb{N}$, respectively, and that t_1 is a total function whereas t_2 is undefined for at least one input $i \in \mathbb{N}$. (We assume that a natural number n is encoded on the tape as a string of n letters 0.)

21		no	*Can it be concluded that there is a primitive recursive function that computes t_1?*

The Ackermann function ack is a total function that is not primitive recursive. Hence, if T_1 is the Turing machine that computes $t_1(n) = \mathrm{ack(n,n)}$, then we can assume that T_1 holds on every input. However, t_1 is not primitive recursive.

22	yes		*Is there a WHILE-program that computes t_2?*

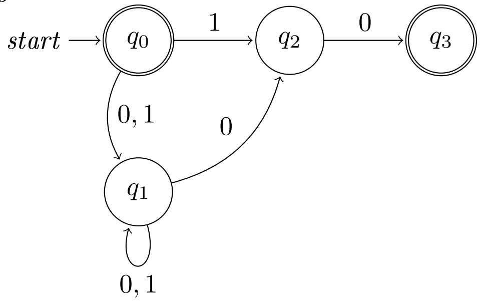
Every Turing machine can be simulated by a WHILE-program.

23	yes		*Is every primitive recursive function computable by a WHILE-program?*

Part 5 *Open2015*

((2 points))

Let $N = (Q, \Sigma, \delta, q_0, F)$ be a nondeterministic finite state machine with $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $S = \{q_0\}$, $F = \{q_0, q_3\}$, and transition function δ as given below.



1. *Let X_i denote the regular expression for the language accepted by N when starting in state q_i.*

   Write down an equation system for $X_0, \ldots, X_3$.

2. *Give a regular expression r such that $L(r) = L(N)$ (you may apply Arden's Lemma to the result of 1).*

$$
\begin{aligned}
X_0 &= (0+1)X_1 + 1X_2 + \varepsilon \\
X_1 &= (0+1)X_1 + 0X_2 \\
X_2 &= 0X_3 \\
X_3 &= \varepsilon \\
r &= \varepsilon + 10 + (0+1)(0+1)^*00
\end{aligned}
$$