Problems Solved:

31	32	33	34	35

Name:

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Problem 31. Show that the Acceptance Problem is reducible to the restricted Halting problem.

Problem 32. Describe (informally) a Turing machine M that generates the universal language

$$L_u = \{\langle M\rangle w \mid M \text{ accepts } w\},$$

i.e., $L_u = Gen(M)$.
You do not have to give an explicit definition of such a machine, but you must clearly describe how such a machine can in principle work, i.e., use higher level constructs to describe the "algorithm" that such a machine represents.

Problem 33. Let $M_0, M_1, M_2, \ldots$ be a list of all Turing machines with alphabet $\Sigma = \{0, 1\}$ such that the function $i \mapsto \langle M_i\rangle$ is computable. Let $w_i = 01^i0$ for all natural numbers i. Let $L = \{w_i \mid i \in \mathbb{N} \text{ and } M_i \text{ accepts } w_i\}$ and $\overline{L} = \Sigma^* \setminus L$.

(a) Is L recursively enumerable?

(b) Is $\overline{L}$ recursively enumerable?

(c) Is L recursive?

(d) Is $\overline{L}$ recursive?

Justify your answers.

Problem 34. Let L be a language over the alphabet $\Sigma = \{0, 1\}$ that is generated by some Turing machine N. For which L is the following problem semi-decidable? For which L is it decidable?
Input of the problem (*instance* of the problem): the code $\langle M\rangle$ of a Turing machine M.
Question of the problem: $L(M) \cap L \neq \emptyset$?

Problem 35. Which of the following problems are decidable? In each problem below, the input of the problem is the code $\langle M\rangle$ of a Turing machine M with input alphabet $\{0, 1\}$.

(a) Does M have at least 4 states?

(b) Is $L(M) \subseteq \{0, 1\}^*$?

(c) Is $L(M)$ recursive?

(d) Is $L(M)$ finite?

(e) Is $10101 \in L(M)$?

(f) Is $L(M)$ not recursively enumerable?

(g) Does there exist a word $w \in L(M)$ such that M does not halt on w?

Justify your answer.