

Dafny

An Automatic Program Verifier for Functional Correctness

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What is Dafny?

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Table of Contents

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- Other Types

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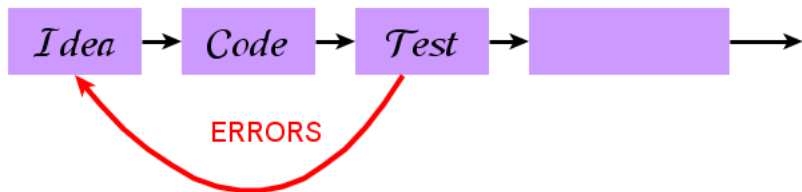
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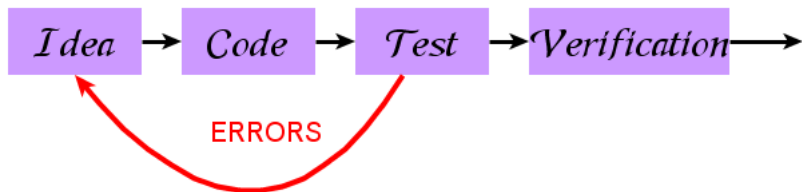
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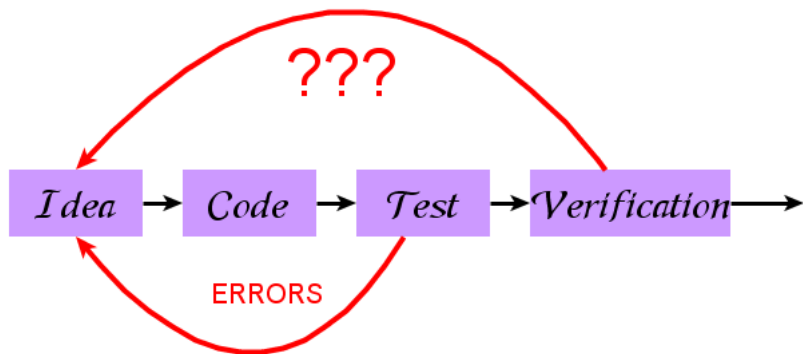
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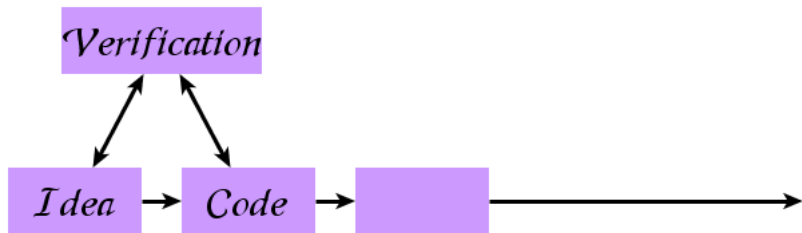


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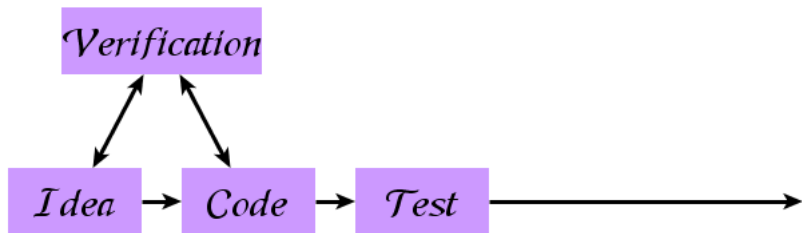
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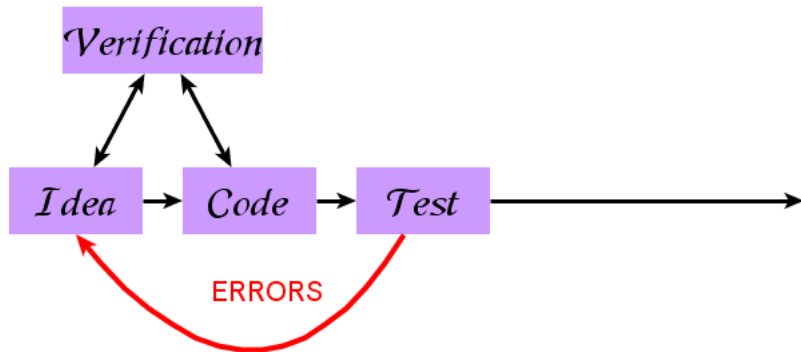
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Development

Dafny was developed by Microsoft Research (MSR) by K. Rustan M. Leino (shorter: Rustan Leino) and others^[2] in 2008.

Founded in 1991, MSR researches among other subjects on the following:

- Algorithms
- Social Computing
- Software Development
- Hardware Development

Some well known experts as Michael Freedman (Mathematician, awarded with Fields-Medal) or Leslie Lamport work or had worked there.

Some developments of MSR are C# and the Windows Sidebar.^[7]

Development

„Dafny started as a little language and verifier to experiment with a certain style of specifications (known as "dynamic frames") for programs that operate on a mutable heap. Since then, Dafny has become a full program verifier with both imperative (assignments, loops, classes, etc.) and functional (datatypes, co-datatypes, higher-order functions, etc.) programming constructs as well as proof authoring facilities (lemmas, proof calculations, refinement, etc.). It has been used for some systems projects, including ExpressOS [ASPLOS 2013], Ironclad Apps [OSDI 2014], and IronFleet [SOSP 2015], and it has been used in various forms of teaching at more than 30 universities.“ - Rustan Leino

Development

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The verification process, (very) briefly summarized:

- ▶ Dafny translates code into Boogie (intermediate verification language)
- ▶ The Boogie code is then verified using the SMT proofer Z3

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Visiting Professor, Department of Computing,
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Homepage:

<http://research.microsoft.com/en-us/um/people/leino/>

Where can I get Dafny?

Dafny can be tried out at <http://rise4fun.com/Dafny/>, where one also can find a very helpful tutorial <http://rise4fun.com/Dafny/tutorial> and examples.

If you want to get more serious, you can download it from <http://dafny.codeplex.com/> and run it from the command line, or, what I actually recommend, running it in Microsoft Visual Studio (2012). I will give a demo of that at the end of this presentation.

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Postcondition and Precondition

Typically, every algorithm has specific Input and Output Conditions! These concept is realized in Dafny by using `requires` and `ensures`.

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method Algorithm1(x:int,y:real) returns (z:nat)
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method ComputeTesseract(x:int) returns (z:int)
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Sometimes, you might see [predicate](#) used instead of [function](#), and the „return type“ missing, as a Predicate is a Function that returns [bool](#). So somehow this is just a shorter way of writing a Function, that corresponds to an attribute. Also, sometimes you might see [function method](#) used, that just means that this is a Function as well as a Method, so you can use the best of both worlds.

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I will give some examples later.

Loop Invariants

Dafny faces a real problem when it reaches the beginning of a loop or a recursion. It has to prove correctness, that means to „go all possible ways“, of a loop. One can easily imagine that this is impossible in general. What it does to solve this problem is, to view the loop as a black box. It is then the programmers duty to provide loop invariants. Dafny then tries to proof that the invariant holds at entering and at every execution of the loop.

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  i:=i+1;
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```

```
var i, a, b := 1,0,1;
while i<n
  invariant 0<=i<=n
  invariant a==fib(i-1)
  invariant b==fib(i)
{
  a, b:=b,a+b;
  i:=i+1;
}
```

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- about $\frac{1}{4}$ of the whole code (including comment),

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- more than $\frac{1}{2}$ of the core loop are code lines for invariants/decreases.

Termination of Loops and Recursions

In some (easy) cases, Dafny can proof termination of loops and recursions without any help. But after programming a few easy recursive Functions or Methods, one will see that Dafny can really prove termination of very little recursions and loops. The programmer then must help Dafny with providing the **decreases** annotation.

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While most of the time this will be an integer, every expression, that:

- 1 Gets smaller and
- 2 Is bounded

can be used.

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Besides the already presented Numeric Types, Dafny also provides Boolean variables. A Boolean value can either be `true` or `false`, which are also the corresponding literals.

With Boolean values and expressions one can use the following operators:

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a && b; a || b; !a;    // AND (conjunction), OR (disjunction), NOT (negation)
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Of course one can also test equality (`==`) and inequality (`!=`) with boolean values and expressions.

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```
predicate haszero(a:set<int>)
{
  exists j :: j in a && (forall i :: i in a ==> j + i == i)
}
```

Sets and Multisets

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Another, really nice, way to generate sets is using the following construct:

```
var s := set x: nat | x < 100 :: isprime (x) == true;
                                must be a function method
```

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Also, one can check if an element is contained within a sequence.

```
assert 2 in s; assert 4 !in s; // These Assertions hold
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Sequences

Another very useful kind of abbreviation is the „Update“ notation. One has to bear in mind though, that a Sequence, as it is immutable, cannot be updated, so instead of saying a sequence is updated, it is more precise to say, that a new sequence, identical up to one element, is created.

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var s:= [1, 1, 2, 3, 4, 5];  
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var s:= [1, 1, 2, 3, 4, 5];  
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```

All these operations work for Arrays as well (when they are transferred into sequences with the slicing operator).

```
var a:= new int[3];  
a[0], a[1], a[2]:=3, 2, 1;  
assert 2 in a[ .. ] // This Assertion holds
```

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[*!] This is planned as future update/feature.

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For Finite Maps one can also calculate the map cardinality

```
ghost var m1 := map [0:=' ', 3:='c', 5:='e', 8:='h', 9:='i'];  
assert |m1| == 5; // This Assertion holds
```

Type Synonyms and Opaque Types

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Such Type Synonyms are, as you would expect, just synonyms, so they have the exact same behavior as the original datatype.

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The reason for declaring such Opaque Types is, that they can be revealed in a module. Using Opaque Types is using higher level of abstractness.

Newtypes and Conversion

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var x: interval := 0.5;  
assert 1 + 3.0 < x;
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```
var euler := 2.71828;  
assert euler.Trunc == 2; // This assertion holds
```

Other Types

Besides the base types (Boolean, Characters and Numerics), the Collection Types and the other presented Types, there are various other Types and Concepts that are known from other programming languages, such as:

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Also, the Functions, I already presented, are seen as Types. The `reads` and `requires` of a Function are Functions themselves.

Table of Contents

1 Motivation

- Why should one use Verification Tools?
- What is Weakest Precondition?
- Development of Dafny
- Where can I get Dafny?

2 Methods and Contracts

- Postcondition and Precondition, Assertions
- Functions and Framing
- Loop Invariants and Termination Metrics

3 Mathematical Language

- Boolean Operators and Quantifiers
- Sets and Multisets
- Sequences and Maps
- Type Synonyms, Opaque Types, Newtypes and Conversion
- Other Types

4 Lemmas and Induction

Lemmas

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This means, one has to „poke“ Dafny into the right direction, to make it consider things it knows but does not consider. This is usually done by a chain of Assertions, Case distinction,...

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