Problems Solved:

36	37	38	39	40

Name:

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Problem 36. Let the *enumerator problem EP* be to decide for a given enumerator M and word w, whether M eventually writes w:

$$EP := \{(\langle M \rangle, w) \mid M \text{writes } w\}$$

Show that *EP* is undecidable by reduction of the *acceptance problem* to *EP*.

Problem 37. Let $M = (Q, \Gamma, \sqcup, \Sigma, \delta, q_0, F)$ be a Turing machine with $Q = \{q_0, q_1\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$, $F = \{q_1\}$ and the following transition function δ:

δ	0	1	$\sqcup$
q_0	$q_0 0 R$	$q_1 1 R$	–
q_1	–	–	–

1. Determine the (worst-case) time complexity $T(n)$ and the (worst-case) space complexity $S(n)$ of M.
2. Determine the average-case time complexity $\bar{T}(n)$ and the average-case space complexity $\bar{S}(n)$ of M. (Assume that all 2^n input words of length n occur with the same probability, i.e., $1/2^n$.)

Note: The summation sign may not be part of the answer and shall be replaced by a closed formula.

Problem 38. True or false?

1. $5n^2 + 7 = O(n^2)$
2. $5n^2 = O(n^3)$
3. $4n + n \log n = O(n)$
4. $(n \log n + 1024 \log n)^2 = O(n^2 (\log n)^3)$
5. $3^n = O(9^n)$
6. $9^n = O(3^n)$

Prove your answers based on the following definition.
Definition: For functions $f, g : \mathbb{N} \to \mathbb{R}_{\geq 0}$ we define

$$g(n) = O(f(n)) \iff \exists c \in \mathbb{R}_{>0} : \exists N \in \mathbb{N} : \forall n \geq N : g(n) \leq c \cdot f(n).$$

Problem 39. Show by formal proof based on the definition of O-notation that for all functions $f, g, h : \mathbb{N} \to \mathbb{R}_{\geq 0}$ the following holds:
If $f = O(g)$ and $g = O(h)$, then $f = O(h)$.

Problem 40. Prove or disprove the following:

1. $O(g(n))^2 = O(g(n)^2)$
2. $2^{O(g(n))} = O(2^{g(n)})$

Hint: First transform above equations into a form that does not involve the O-notation on the left hand side, then prove the correctness of the resulting formulas.