**Problems Solved:**

| 31 | 32 | 33 | 34 | 35 |
|---|---|---|---|---|

**Name:**

**Matrikel-Nr.:**

**Problem 31.** Let $\Sigma$ be an alphabet and $A$ be a set ($A \subseteq \Sigma^*$). Let also $A$ be semi-decidable, but not decidable. Prove that the complement of $A$ is not decidable.

**Problem 32.** Let $M_0, M_1, M_2, \ldots$ be a list of all Turing machines with alphabet $\Sigma = \{0,1\}$ such that the function $i \mapsto \langle M_i \rangle$ is computable. Let $w_i = 01^i0$ for all natural numbers $i$. Let $L = \{w_i \mid i \in \mathbb{N} \text{ and } M_i \text{ accepts } w_i\}$ and $\overline{L} = \Sigma^* \setminus L$.

(a) Is $L$ recursively enumerable?

(b) Is $\overline{L}$ recursively enumerable?

(c) Is $L$ recursive?

(d) Is $\overline{L}$ recursive?

Justify your answers.

**Problem 33.** Let $L$ be a finite language over an alphabet $\{0,1\}$. Is the following problem (with input $\langle M \rangle$)

> For a Turing maschine $M$ it holds $L(M) \supseteq L$.

in general semi-decidable? Is it also in general decidable?

**Problem 34.** Which of the following problems are decidable? In each problem below, the input of the problem is the code $\langle M \rangle$ of a Turing machine $M$ with input alphabet $\{0,1\}$.

1. Is $L(M)$ empty?
2. Is $L(M)$ finite?
3. Is $L(M)$ regular?
4. Is $L(M) \subseteq \{0,1\}^*$?
5. Is $L(M)$ not recursively enumerable?
6. Does $M$ have an even number of states?

**Problem 35.** Let $M$ be a Turing maschine with the following property: *If $M$ accepts a word, then this is done in less than 1000 steps.*

1. Is $L(M)$ recursively enumerable?
2. Is $L(M)$ recursive?
3. Is the property of $L(M)$ to contain the empty word, decidable?
4. Is $L(M)$ necessarily finite?