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Klausur 1

Berechenbarkeit und Komplexität

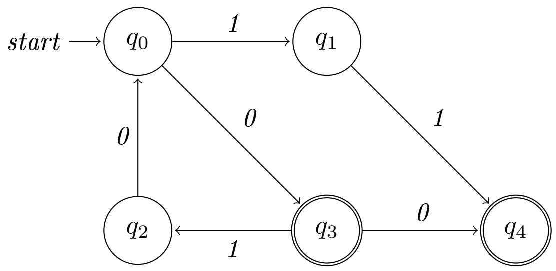
21. November 2014

Part 1 *NFSM2014*

Let N be the nondeterministic finite state machine

$$(\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \nu, \{q_0\}, \{q_3, q_4\}),$$

whose transition function ν is given below.



1		no

Is $0100100100101 \in L(N)$*?*

A word $w \in L(N)$ with $|w| > 1$ never ends with 01.

2	yes	

Let N' be the NFSM that is constructed from N by solely reversing the arrow $q_3 \to q_2$ in the diagram above. Is $L(N')$ finite?

$L(N') = \{0, 00, 11\}$.

3	yes	

Does there exist a regular expression r such that $L(r) = \overline{L(N)} = \{0,1\}^* \setminus L(N)$?

$L(N)$ is regular and so is its complement.

4		no
5	yes	
6		no

4. *Let $L = \{\, 1^n w \mid n \in \mathbb{N}, w \in L(N), n = |w| \,\}$. Is L a regular language?*

5. *Is there an enumerator Turing machine G such that $Gen(G) = L(N)$?*

6. *Let $M = (Q, \Gamma, \sqcup, \{0,1\}, \delta, q, F)$ be a deterministic Turing machine such that $L(M) = L(N)$ and let $M' = (Q, \Gamma, \sqcup, \{0,1\}, \delta, q, Q \setminus F)$. Can one conclude that M' halts on every word that is not in $L(N)$.*

$1 \notin L(N)$. Take a Turing machine M with $L(M) = L(N)$ that runs forever on the input 1, i.e., there is always a following configuration. So $1 \notin L(M)$. But for the same reason $1 \notin L(M')$.

7	yes	

Is there a deterministic Turing machine $T = (Q, \Gamma, \sqcup, \{0,1\}, \delta, q, F)$ with $L(T) = L(N)^*$?

$L(N)$ is regular. Hence, $L(N)^*$ is regular, and thus also recursively enumerable.

8	yes	

Let L be an arbitrary language that contains only finitely many words. For each Turing machine T, does there exist a deterministic finite state machine D such that $L(D) = L \cap L(T)$.

Since L is finite, so is $L \cap L(T)$ and therefore, regular.

Part 2 *Computable2014*

We know that there exists a Turing machine that enumerates the decimal expansion of π, i.e., that writes the infinite sequence 3 1 4 1 5 9 2 6 5 ... on its tape. Let $\varphi : \mathbb{N} \to \mathbb{N}$ be the function such that $\varphi(n)$ is the n^{th} digit in the decimal expansion of π. Let $\varrho : \mathbb{N} \times \mathbb{N} \to \{0,1\}$ be defined by

$$\varrho(n,m) = \begin{cases} 0, & \text{if } \varphi(n) = m, \\ 1, & \text{otherwise.} \end{cases}$$

9	yes		*Is φ Turing-computable?*
10	yes		*Is ϱ Turing-computable?*
11	yes		*Is $\varphi(1000000)$ loop-computable?* Hint: *a number is loop computable, if there exists a loop program that writes into x0 this number.*
12		no	*For every WHILE-program W, does there exist a Turing machine T that halts on every input such that W and T compute the same function?*

Let f be the (partial) function that is only defined for even numbers, say, $f(n) = n$ if $n \in \mathbb{N}$ even and $f(n)$ is undefined for odd n. Let W be the WHILE-program that computes f. Since a Turing machine T that halts on every input computes a total function, it surely does not compute f.

13	yes		*Is every Turing-computable function also a μ-recursive function?*
14		no	*Does there exist a μ-recursive function that is not WHILE computable?*

Part 3 *Pumping2014*

Let

$$L_1 = \left\{ 0^{(n^2)} 1^n \,\middle|\, n \in \mathbb{N}, n < 98765 \right\} \subset \{0,1\}^*,$$
$$L_2 = \{ 0^m 1^n \mid m,n \in \mathbb{N}, m > n > 1 \} \subset \{0,1\}^*.$$

15	yes		*Is there a regular expression r such that $L(r) = \overline{L_1} := \{0,1\}^* \setminus L_1$?*

L_1 is regular, i.e., its complement $\overline{L_1}$ is also regular.

16		no	*Is there a deterministic finite state machine M such that $L(M) = L_2$?*
17	yes		*Is there an enumerator Turing machine G such that $Gen(G) = L_1$?*
18	yes		*Is there an deterministic finite state machine D such that $L(D) = L_1 \cap L_2$?*

The language $L_1 \cap L_2$ is finite and thus regular.

19	yes		*Are there two languages X_1 and X_2 that are not regular, but for which $X_1 \cup X_2$ is regular?*

Yes. Take $X_1 = L_2$ and $X_2 = \overline{L_2}$.

Part 4 *WhileLoop2014*

Let T_1 and T_2 be two Turing machines. Assume that T_1 and T_2 compute partial functions $t_1, t_2 : \mathbb{N} \to \mathbb{N}$, respectively, and that t_1 is a total function whereas t_2 is undefined for at least one input $i \in \mathbb{N}$. (We assume that a natural number n is encoded on the tape as a string of n letters 0.)

20		no

Can it be concluded that t_1 is LOOP-computable?

The Ackermann function ack is a total function that is not primitive recursive. Hence, if T_1 is the Turing machine that computes $t_1(n) = \mathrm{ack}(\mathrm{n}, \mathrm{n})$, then we can assume that T_1 holds on every input. However, since t_1 is not primitive recursive, there cannot be a corresponding LOOP-program.

21	yes	

Is there a WHILE-program that computes t_2?

Every Turing machine can be simulated by a WHILE-program.

22	yes	

Is the composition $t_1 \circ t_2$ a μ-recursive function?
Hint: $(t_1 \circ t_2)(x) = t_1(t_2(x))$.

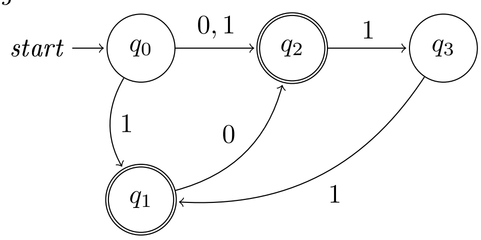
23	yes	

Is every primitive recursive function computable by a LOOP-program?

Part 5 *Open2014*

((2 points))

Let $N = (Q, \Sigma, \delta, q_0, F)$ be a nondeterministic finite state machine with $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $S = \{q_0\}$, $F = \{q_1, q_2\}$, and transition function δ as given below.



1. *Let X_i denote the regular expression for the language accepted by N when starting in state q_i.*

   Write down an equation system for $X_0, \ldots, X_3$.

2. *Give a regular expression r such that $L(r) = L(N)$ (you may apply Arden's Lemma to the result of 1).*

$$
\begin{aligned}
X_0 &= 1X_1 + (0+1)X_2 \\
X_1 &= 0X_2 + \varepsilon \\
X_2 &= 1X_3 + \varepsilon \\
X_3 &= 1X_1 \\
r &= 0 + 1 + (1 + (0+1)11)(011)^*(0 + \varepsilon)
\end{aligned}
$$