Problems Solved:

26	27	28	29	30

Name:

Matrikel-Nr.:

Problem 26. Let $f : \mathbb{N} \to \mathbb{N}$ be the (partial) function

$$f(x) = \min_y (x \leq y^2)$$

1. Is f a μ-recursive function?
2. Is f a while computable function?
3. Is f a primitive recursive function?
4. Is f a loop computable?

In each case justify your answer. If it is *yes*, give a corresponding definition. Remark: When defining f, you are allowed to use the primitive recursive functions (respectively loop programs computing these functions)

$$m : \mathbb{N}^2 \to \mathbb{N}, \quad (x, y) \mapsto x \cdot y$$

and $u : \mathbb{N}^2 \to \mathbb{N}$,

$$u(x, y) = \begin{cases} 0 & \text{if } x = y, \\ 1 & \text{if } x \neq y. \end{cases}$$

Problem 27. Consider the following term rewriting system:

$$p(x, s(y)) \to p(s(x), y) \tag{1}$$
$$p(x, 0) \to x \tag{2}$$

1. Show that
$$p(s(0), s(0)) \xrightarrow{*} s(s(0))$$
by a suitable reduction sequence. For each reduction step, underline the subterm that you reduce, and indicate the reduction rule and the matching substitution σ used explicitly.

2. Disprove that
$$p(p(s(0), s(0)), p(s(0), s(0))) \xrightarrow{*} s(s(0)).$$

Problem 28. Define the following languages by context-free grammars over the alphabet $\Sigma = \{0, 1\}$.

(a) $L_1 = \{w \mid w \text{ contains at least two zeroes.}\}$

(b) $L_2 = \{w \mid w \text{ starts and ends with one and the same symbol.}\}$

(c) $L_3 = \{w \mid w \text{ consists of an odd number of symbols and the symbol in the center of } w \text{ is a } 0.\}$

(d) $L_4 = L_2 \cap L_3$

Problem 29. Consider the grammar $G = (N, \Sigma, P, S)$ where $N = \{S\}$, $\Sigma = \{a, b\}$, $P = \{S \to \varepsilon, S \to aSbS\}$.

(a) Is $aababb \in L(G)$?

(b) Is $aabab \in L(G)$?

(c) Does every element of $L(G)$ contain the same number of occurrences of a and b?

(d) Is $L(G)$ regular?

(e) Is $L(G)$ recursive?

Justify your answers.

Problem 30. Construct a FSM recognizing $L(G)$ where G is the grammar:

$$S \to aS|bA|b$$

$$A \to aA|bS|a$$