Problems Solved:

16	17	18	19	20

Name:

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Problem 16. Write down explicitly an enumerator G such that $\mathrm{Gen}(G) = \{0^{2n} \mid n \in \mathbb{N}\}$.
Since in the lecture notes it has not been *formally* defined, how a Turing machine with two tapes works, you may describe the transition function as

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, L\} \times (\Gamma \cup \{\boxtimes\})$$

in the following way: If G is in state q and reads the symbol c from the working tape, and

$$\delta(q, c) = (q', c', d, c'')$$

then G goes to state q', replaces c by c' on the working tape and moves the working tape head in direction d. Moreover, unless $c'' = \boxtimes$, the symbol c'' is written on the output tape and the output tape head is moved one position forward. If, however, $c'' = \boxtimes$, nothing is written on the output tape and the output tape head rests in place.
Hint: There exists a solution with only 4 states.

Problem 17. Give reasons for your answers. And think about the implications for the general computability of functions.

1. Let R be a RAM that reads exactly one number from its input tape and always terminates with 0 or 1 written on its output tape. Is there a function $f : \mathbb{Z} \to \mathbb{Z}$ such that $f(x) = y$ if and only if the input was x and after termination y is on the output tape of R?

2. Let $f : \mathbb{Z} \to \mathbb{Z}$ be a function. Is there always a RAM R such that R terminates on every input and that R with input $z \in \mathbb{Z}$ has written $f(z)$ to its output tape?

Hint: The task basically asks you whether there is a bijection between the set of functions from $\mathbb{Z}$ to $\mathbb{Z}$ and the set of all RAMs. For one part you can use a cardinality argument for those sets.

Problem 18. Let $\Sigma = \{a, b\}$. We code a and b on the input tape of a RAM by 1 and 2 and a word $w \in \Sigma^*$ by a respective sequence of 1's and 2's.
We say that a RAM R *accepts* a word $w \in \Sigma^*$ if R starts with the coded word w on its input tape and terminates after having written a non-zero number on its output tape. We define $L(R) := \{w \in \Sigma^* \mid R \text{ accepts } w\}$.
Let F be a RAM that terminates for every input and whose program does not contain "loops", i.e., each instruction is executed at most once.
Derive answers for the following questions. (Give ample justifications, just saying 'yes' or 'no' is not enough.)

1. Is $L(F)$ as a language over Σ finite?

2. Is $L(F)$ as a language over Σ regular?

Problem 19. Provide a loop program that computes the function $f(n) = n! = \prod_{k=1}^{n} k$, and thus show that f is loop computable. In your program you are only allowed to uses statements from Definition 23 of the lecture notes.

Problem 20. Suppose P is a while-program that does not contain any `WHILE` statements, but might modify the values of the variables x_1 and x_2.
Transform the following program into Kleene's normal form.
Hint: first translate the program into a goto program, replace the `GOTO`s by assignments to a control variable, and add the `WHILE` wrapper.

```
x0 := 0
WHILE x1 DO
    x1 := x1 - 1;
    x2 := x1;
    WHILE x2 DO
        P;
    END;
END;
x0 := x0 + 1
```