

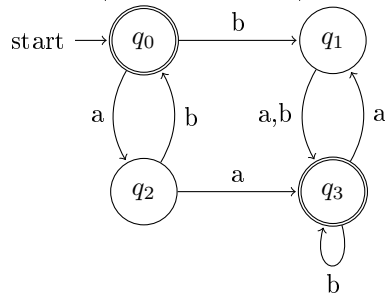
Problems Solved:

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| 11 | 12 | 13 | 14 | 15 |
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Name:**Matrikel-Nr.:****Problem 11.** Answer the following questions.

- (a) Is the language $L = \{0^m 1^n \mid m, n \in \mathbb{N}\}$ regular?
- (b) Is the language $L = \{0^n 1^n \mid n \in \mathbb{N}\}$ regular?
- (c) Is every subset of a regular language again a regular language?

Problem 12. Let M_1 be the DFSM with states $\{q_1, q_2, q_3, q_4\}$ whose transition graph is given below. Determine a regular expression r such that $L(r) = L(M_1)$. Show the *derivation* of the the final result by the technique based on Arden's Lemma (see lecture notes).

**Problem 13.** Let r be the following regular expression.

$$a \cdot a \cdot (b \cdot a)^* \cdot b \cdot b^*$$

Construct a nondeterministic finite state machine N such that $L(N) = L(r)$. Show the derivation of the result by following the technique presented in the proof of the theorem *Equivalence of Regular Expressions and Automata* (see lecture notes).

Problem 14. Let L be the language of properly nested strings over the alphabet $\Sigma = \{[,], \circ\}$. A word w is *properly nested* if it contains as many opening as closing brackets and every prefix of w contains at least as many opening brackets $[$ as closing $]$. (Example: $\circ\circ[[][\circ[\circ]]]$ is properly nested, but $\circ\circ][[$ is not.) Show by means of the Pumping Lemma that L is not regular.

Problem 15. Write down explicitly a Turing machine M over $\Sigma = \{0\}$ which computes the function $d : \mathbb{N} \rightarrow \mathbb{N}$ given by $d(n) = 2n$. Use unary representation: A number n is represented by the string 0^n consisting of n copies of the symbol 0.