Problems Solved:

11	12	13	14	15

Name:

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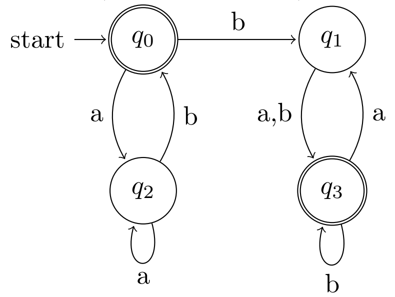
Problem 11. Let M_1 be the DFSM with states $\{q_1, q_2, q_3, q_4\}$ whose transition graph is given below. Determine a regular expression r such that $L(r) = L(M_1)$. Show the *derivation* of the the final result by the technique based on Arden's Lemma (see lecture notes).



Problem 12. Let r be the following regular expression.

$$a \cdot a \cdot (b \cdot a)^* \cdot b \cdot b^*$$

Construct a nondeterministic finite state machine N such that $L(N) = L(r)$. Show the derivation of the result by following the technique presented in the proof of the theorem *Equivalence of Regular Expressions and Automata* (see lecture notes).

Problem 13. Let L be the language of properly nested strings over the alphabet $\Sigma = \{\texttt{[}, \texttt{]}, \texttt{o}\}$. A word w is *properly nested* if it contains as many opening as closing brackets and every prefix of w contains at least as many opening brackets `[` as closing `]`. (Example: `oo[][o[o]]` is properly nested, but `oo][` is not.) Show by means of the Pumping Lemma that L is not regular.

Problem 14. Write down explicitly a Turing machine M over $\Sigma = \{0\}$ which computes the function $d : \mathbb{N} \to \mathbb{N}$ given by $d(n) = 2n$.
Use unary representation: A number n is represented by the string 0^n consisting of n copies of the symbol 0.

Problem 15. Write down explicitly an enumerator G such that $\text{Gen}(G) = \{0^{2n} \mid n \in \mathbb{N}\}$.
Since in the lecture notes it has not been *formally* defined, how a Turing machine with two tapes works, you may describe the transition function as

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, L\} \times (\Gamma \cup \{\boxtimes\})$$

in the following way: If G is in state q and reads the symbol c from the working tape, and

$$\delta(q, c) = (q', c', d, c'')$$

then G goes to state q', replaces c by c' on the working tape and moves the working tape head in direction d. Moreover, unless $c'' = \boxtimes$, the symbol c'' is

written on the output tape and the output tape head is moved one position forward. If, however, $c'' = \boxtimes$, nothing is written on the output tape and the output tape head rests in place.
Hint: There exists a solution with only 4 states.