Problems Solved:

1	2	3	4	5

Name:

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Problem 1. Show by induction that for all real numbers $a \notin \{0, 1, -1\}$ the following holds:

$$\forall n \in \mathbb{N} : \quad \sum_{k=0}^{n} a^{2k} = \frac{a^{2n+2} - 1}{a^2 - 1}.$$

Problem 2. Let $L \subseteq \Sigma^*$ be a language over the alphabet $\Sigma = \{a, b, c, d\}$ such that a word w is in L if and only if it is either a or b or of the form $w = ducvd$ where u and v are words of L. For example, *dacad*, *ddacbdcad*, *dddbcbdcdbcbddcad* are words in L. Show by induction that every word of L contains an even number of the letter d.
Note that a *language* is just a set of words and a *word* is simply a finite sequence of letters from the alphabet.

Problem 3. Show $\sqrt[3]{5} \notin \mathbb{Q}$ by an indirect proof.
Hint: `http://en.wikipedia.org/wiki/Square_root_of_2#Proofs_of_irrationality`.

Problem 4. Let L be the lanugage over $\{0, 1\}$ that consists of binary representations of non-negative multiples of 3. The set L is such that if $w \in L$ then also $0w \in L$, in other words, leading zeroes in representations are OK. For example, 0, 00, 11, 001001 are in L, but 1, 010 are not.
Construct a deterministic finite-state machine $M = (Q, \Sigma, \delta, q_0, F)$ over the alphabet $\{0, 1\}$ such that $L = L(M)$. Draw the transition graph of M.
Hint: For a word $w \in L$, let $V(w)$ be the number corresponding to the binary representation given by the word w. Let the states of M correspond to $V(w)$ mod 3 and note that $V(\varepsilon) = 0$, $V(w0) = 2V(w)$ and $V(w1) = 2V(w) + 1$.

Problem 5. Construct a deterministic finite state machine M over $\Sigma = \{0, 1\}$ such that $L(M)$ consists of all words that do not contain the string 01. *Hint:* Start by constructing a nondeterministic finite state machine N that recogizes the words that *do* contain the string 01. Proceed by converting your nondeterministic machine N to a deterministic machine D that accepts the same language. Now you are left with the task of coming up with a machine M whose language is precisely the complement of the language of D. This can be done by a small modification of D.