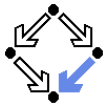
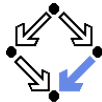


Computer-Supported Program Verification with the RISC ProofNavigator

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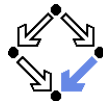
1. An Overview of the RISC ProofNavigator

2. Specifying Arrays

3. Verifying the Linear Search Algorithm

4. Conclusions

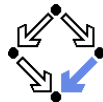
The RISC ProofNavigator



- **An interactive proving assistant for program verification.**
 - Research Institute for Symbolic Computation (RISC), 2005–:
<http://www.risc.jku.at/research/formal/software/ProofNavigator>.
 - Development based on prior experience with PVS (SRI, 1993–).
 - Kernel and GUI implemented in Java.
 - Uses external SMT (satisfiability modulo theories) solver.
 - CVCL (Cooperating Validity Checker Lite) 2.0.
 - Runs under Linux (only); freely available as open source (GPL).
- **A language for the definition of logical theories.**
 - Based on a strongly typed higher-order logic (with subtypes).
 - Introduction of types, constants, functions, predicates.
- **Computer support for the construction of proofs.**
 - Commands for basic inference rules and combinations of such rules.
 - Applied interactively within a sequent calculus framework.
 - Top-down elaboration of proof trees.

Designed for simplicity of use; applied to non-trivial verifications.

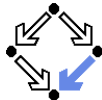
Using the Software



For survey, see “Program Verification with the RISC ProofNavigator”.
For details, see “The RISC ProofNavigator: Tutorial and Manual”.

- **Develop a theory.**
 - Text file with declarations of types, constants, functions, predicates.
 - Axioms (propositions assumed true) and formulas (to be proved).
- **Load the theory.**
 - File is read; declarations are parsed and type-checked.
 - Type-checking conditions are generated and proved.
- **Prove the formulas in the theory.**
 - Human-guided top-down elaboration of proof tree.
 - Steps are recorded for later replay of proof.
 - Proof status is recorded as “open” or “completed”.
- **Modify theory and repeat above steps.**
 - Software maintains dependencies of declarations and proofs.
 - Proofs whose dependencies have changed are tagged as “untrusted”.

Starting the Software



- Starting the software:

ProofNavigator & (32 bit machines at RISC)

ProofNavigator64 & (64 bit machines at RISC)

- Command line options:

Usage: ProofNavigator [OPTION]... [FILE]

FILE: name of file to be read on startup.

OPTION: one of the following options:

-n, --nogui: use command line interface.

-c, --context NAME: use subdir NAME to store context.

--cvcl PATH: PATH refers to executable "cvcl".

-s, --silent: omit startup message.

-h, --help: print this message.

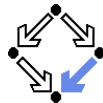
- Repository stored in subdirectory of current working directory:

ProofNavigator/

- Option -c *dir* or command newcontext "*dir*" :

- Switches to repository in directory *dir*.

The Graphical User Interface



The screenshot shows the RISC ProofNavigator interface. The window title is "RISC ProofNavigator".

Proof Tree (Left Panel):

- [dca]: expand invariant, Output
- [bvj]: scatter
 - [dca]: auto
 - [t4c]: proved (CVCL)
 - [scu]: split plq
 - [kal]: proved (CVCL)
 - [lal]: scatter
 - [lvn] (highlighted)
 - [fou]
 - [gou]: proved (CVCL)

Proof State (Right Panel):

Formula [C] proof state [lvn]

Constants (with types): anyelem, r, get, length, put, Invariant, content, j₀, anyarray, new, Output, Input, oldx, i, a, n, olda, any, x.

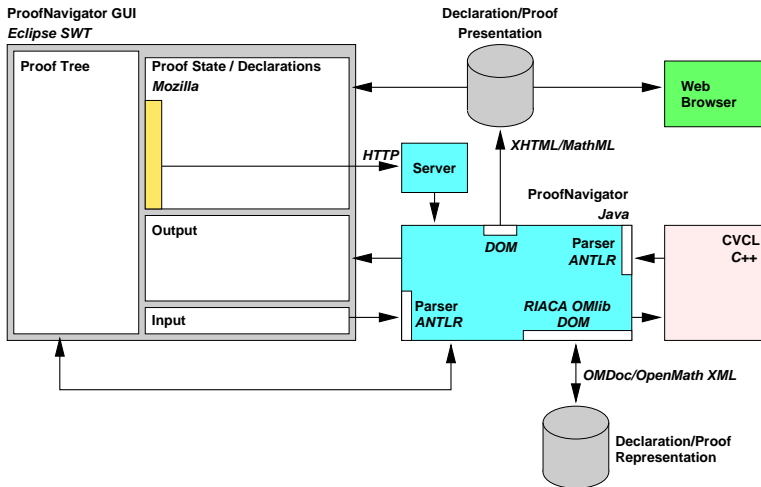
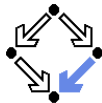
```
ed2 olda = a
cmz oldx = x
hvv n = length(a)
564  $\forall j \in \mathbb{N}: x = \text{get}(a, j) \Rightarrow j \geq i$ 
mys i  $\leq n$ 
gkr  $r = -1 \vee r = i \wedge x = \text{get}(a, r) \wedge i < n$ 
orv  $r = -1 \Rightarrow n \leq i$ 
k4w  $x = \text{get}(a, j_0)$ 
6ha  $j_0 < n$ 
jns  $0 \leq r$ 
```

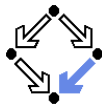
View Declarations

Input/Output

```
ELEM], arrayelem ([NAT, ARRAY NAT OF ELEM], ELEM, NAT, NAT, INT)-BOOLEAN, CONTENT, INVT,
ARRAY NAT OF ELEM]-ARRAY NAT OF ELEM, j_0: NAT, anyarray: [NAT, ARRAY NAT OF ELEM], new: NAT-
->[NAT, ARRAY NAT OF ELEM], Output: BOOLEAN, Input: BOOLEAN, olda: ELEM, i: NAT, a: [NAT,
ARRAY NAT OF ELEM], n: NAT, olda: [NAT, ARRAY NAT OF ELEM], any: ARRAY NAT OF ELEM, x: ELEM,
[ed2] olda = a
[cmz] oldx = x
[hvv] n = length(a)
[564] FORALL(j:NAT): x = get(a, j) => j >= i
[mys] i <= n
[gkr] r = -1 OR r = i AND x = get(a, r) AND i < n
[orv] r = -1 => n <= i
[k4w] x = get(a, j_0)
[6ha] j_0 < n
.....
[jns] 0 <= r
prove>
```

The Software Architecture



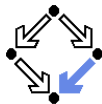


Software Components

- **Graphical user interface.**
 - Display of declarations and proof state.
 - Embeds HTML browser as core component.
- **Proof engine.**
 - Commands for navigating the proof.
 - Interaction with validity checker to simplify/close proof states.
- **Validity checker.**
 - Simplifies formulas
 - Checks the validity of formulas.
 - Produces counterexamples for (presumably) invalid formulas.
- **Object repository.**
 - Proof persistence.
 - Proof status management.

All data are externally represented in (gzipped) XML.

A Theory

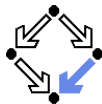


```
% switch repository to "sum"
newcontext "sum";

% the recursive definition of the sum from 0 to n
sum: NAT->NAT;
S1: AXIOM sum(0)=0;
S2: AXIOM FORALL(n:NAT): n>0 => sum(n)=n+sum(n-1);

% proof that explicit form is equivalent to recursive definition
S: FORMULA FORALL(n:NAT): sum(n) = (n+1)*n/2;
```

Declarations written with an external editor in a text file.



Proving a Formula

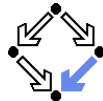
When the file is loaded, the declarations are pretty-printed:

```
sum ∈ ℕ → ℕ
axiom S1 ≡ sum(0) = 0
axiom S2 ≡ ∀ n ∈ ℕ : n > 0 ⇒ sum(n) = n + sum(n - 1)
S ≡ ∀ n ∈ ℕ : sum(n) =  $\frac{(n+1) \cdot n}{2}$ 
```

The proof of a formula is started by the `prove` command.

	Formula S
<code>prove S:</code>	Construct Proof
<code>proof S:</code>	Show Proof
<code>formula S:</code>	Print Formula

Proving a Formula



RISC ProofNavigator
File Options Help

Proof Tree
[tca]

Proof State
Formula [S] proof state [tca]

Constants (with types): sum.

[lxe] $\forall n \in \mathbb{N}: n > 0 \Rightarrow \text{sum}(n) = n + \text{sum}(n-1)$

[d3] $\text{sum}(0) = 0$

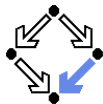
[byu] $\forall n \in \mathbb{N}: \text{sum}(n) = \frac{(n+1)n}{2}$

View Declarations

Input/Output

```
read "sum.pn";
Value sum: NAT->NAT.
Formula S1.
Formula S2.
Formula S.
File sum.pn read.
prove S.
Proof of formula S.
Proof state [tca]
Constants: sum: NAT->NAT.
[lxe] FORALL(n:NAT): n > 0 => sum(n) = n+sum(n-1)
[d3i] sum(0) = 0
-----
[byu] FORALL(n:NAT): sum(n) = (n+1)*n/2
prove>
```

Proving a Formula



- Proof of formula F is represented as a **tree**.
 - Each tree node denotes a **proof state (goal)**.
 - Logical sequent:
 $A_1, A_2, \dots \vdash B_1, B_2, \dots$
 - Interpretation:
 $(A_1 \wedge A_2 \wedge \dots) \Rightarrow (B_1 \vee B_2 \vee \dots)$
 - Initially single node $Axioms \vdash F$.

Constants: $x_0 \in S_0, \dots$

$[L_1] \quad A_1$

\dots

$[L_n] \quad A_n$

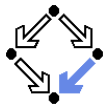
$[L_{n+1}] \quad B_1$

\dots

$[L_{n+m}] \quad B_m$

- The **tree must be expanded to completion**.
 - Every leaf must denote an obviously valid formula.
 - Some A_i is false or some B_j is true.
- A proof step consists of the **application of a proving rule to a goal**.
 - Either the goal is recognized as true.
 - Or the goal becomes the parent of a number of children (subgoals).
The conjunction of the subgoals implies the parent goal.

An Open Proof Tree



Proof Tree

▾ [tca]: induction n in byu

[dbj]: proved (CVCL)

[ebj]

Formula [S] proof state [dbj]

Constants (with types): sum.

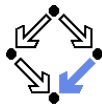
[xe] $\forall n \in \mathbb{N}: n > 0 \Rightarrow \text{sum}(n) = n + \text{sum}(n-1)$

[d3i] $\text{sum}(0) = 0$

[nfq] $\text{sum}(0) = \frac{(0+1) \cdot 0}{2}$

Parent: [tca]

Closed goals are indicated in blue; goals that are open (or have open subgoals) are indicated in red. The red bar denotes the “current” goal.



A Completed Proof Tree

Proof Tree





- ▾ [tca]: induction n in byu
 - [dbj]: proved (CVCL)
- ▾ [ebj]: instantiate n_0+1 in lxe
 - [k5f]: proved (CVCL)

The visual representation of the complete proof structure; by clicking on a node, the corresponding proof state is displayed.

Navigation Commands

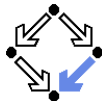


Various buttons support navigation in a proof tree.






- : prev
 - Go to previous open state in proof tree.
- : next
 - Go to next open state in proof tree.
- : undo
 - Undo the proof command that was issued in the parent of the current state; this discards the whole proof tree rooted in the parent.
- : redo
 - Redo the proof command that was previously issued in the current state but later undone; this restores the discarded proof tree.

Single click on a node in the proof tree displays the corresponding state;
double click makes this state the current one.

Proving Commands

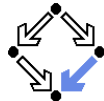


The most important proving commands can be also triggered by buttons.

-  (scatter)
 - Recursively applies decomposition rules to the current proof state and to all generated child states; attempts to close the generated states by the application of a validity checker.
-  (decompose)
 - Like scatter but generates a single child state only (no branching).
-  (split)
 - Splits current state into multiple children states by applying rule to current goal formula (or a selected formula).
-  (auto)
 - Attempts to close current state by instantiation of quantified formulas.
-  (autostar)
 - Attempts to close current state and its siblings by instantiation.

Automatic decomposition of proofs and closing of proof states.

Proving Commands

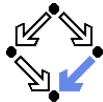


More commands can be selected from the menus.





- **assume**
 - Introduce a new assumption in the current state; generates a sibling state where this assumption has to be proved.
- **case:**
 - Split current state by a formula which is assumed as true in one child state and as false in the other.
- **expand:**
 - Expand the definitions of denoted constants, functions, or predicates.
- **lemma:**
 - Introduce another (previously proved) formula as new knowledge.
- **instantiate:**
 - Instantiate a universal assumption or an existential goal.
- **induction:**
 - Start an induction proof on a goal formula that is universally quantified over the natural numbers.

Here the creativity of the user is required!

Auxiliary Commands

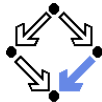


Some buttons have no command counterparts.

- : counterexample
 - Generate a “counterexample” for the current proof state, i.e. an interpretation of the constants that refutes the current goal.
- 
 - Abort current prover activity (proof state simplification or counterexample generation).
- 
 - Show menu that lists all commands and their (optional) arguments.
- 
 - Simplify current state (if automatic simplification is switched off).

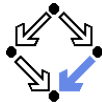
More facilities for proof control.

Proving Strategies



- Initially: semi-automatic proof decomposition.
 - `expand` expands constant, function, and predicate definitions.
 - `scatter` aggressively decomposes a proof into subproofs.
 - `decompose` simplifies a proof state without branching.
 - `induction` for proofs over the natural numbers.
- Later: critical hints given by user.
 - `assume` and `case cut` proof states by conditions.
 - `instantiate` provide specific formula instantiations.
- Finally: simple proof states are yielded that can be automatically closed by the validity checker.
 - `auto` and `autostar` may help to close formulas by the heuristic instantiation of quantified formulas.

Appropriate combination of semi-automatic proof decomposition, critical hints given by the user, and the application of a validity checker is crucial.



-
1. An Overview of the RISC ProofNavigator
 - 2. Specifying Arrays**
 3. Verifying the Linear Search Algorithm
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A Constructive Definition of Arrays



```
% constructive array definition
newcontext "arrays2";

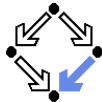
% the types
INDEX: TYPE = NAT;
ELEM:  TYPE;
ARR:   TYPE =
  [INDEX, ARRAY INDEX OF ELEM];

% error constants
any:    ARRAY INDEX OF ELEM;
anyelem: ELEM;
anyarray: ARR;

% a selector operation
content:
  ARR -> (ARRAY INDEX OF ELEM) =
    LAMBDA(a:ARR): a.1;

% the array operations
length: ARR -> INDEX =
  LAMBDA(a:ARR): a.0;
new: INDEX -> ARR =
  LAMBDA(n:INDEX): (n, any);
put: (ARR, INDEX, ELEM) -> ARR =
  LAMBDA(a:ARR, i:INDEX, e:ELEM):
    IF i < length(a)
      THEN (length(a),
            content(a) WITH [i]:=e)
      ELSE anyarray
    ENDIF;
get: (ARR, INDEX) -> ELEM =
  LAMBDA(a:ARR, i:INDEX):
    IF i < length(a)
      THEN content(a)[i]
      ELSE anyelem
    ENDIF;
```

Proof of Fundamental Array Properties



% the classical array axioms as formulas to be proved

length1: FORMULA

FORALL(n:INDEX): length(new(n)) = n;

length2: FORMULA

FORALL(a:ARR, i:INDEX, e:ELEM):

i < length(a) => length(put(a, i, e)) = length(a);

get1: FORMULA

FORALL(a:ARR, i:INDEX, e:ELEM):

i < length(a) => get(put(a, i, e), i) = e;

get2: FORMULA

FORALL(a:ARR, i, j:INDEX, e:ELEM):

i < length(a) AND j < length(a) AND

i /= j =>

get(put(a, i, e), j) = get(a, j);

[adu]: expand length, get, put, content

[c3b]: scatter

[qid]: proved (CVCL)

Proof of a Higher-Level Array Property



```
% extensionality on low-level arrays
```

```
extensionality: AXIOM
```

```
FORALL(a, b:ARRAY INDEX OF ELEM):  
  a=b <=> (FORALL(i:INDEX):a[i]=b[i]);
```

```
% unassigned parts hold identical values
```

```
unassigned: AXIOM
```

```
FORALL(a:ARR, i:INT):  
  (i >= length(a)) => content(a)[i
```

```
[adt]: expand length, get, content
```

```
[cw2]: scatter
```

```
[qey]: proved (CVCL)
```

```
[rey]: assume b_0.1 = a_0.1
```

```
[zpt]: proved (CVCL)
```

```
[1pt]: instantiate a_0.1, b_0.1 in 1fm
```

```
[y51]: scatter
```

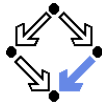
```
[ku2]: auto
```

```
[iub]: proved (CVCL)
```

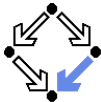
```
% extensionality on arrays to be proved
```

```
equality: FORMULA
```

```
FORALL(a:ARR, b:ARR): a = b <=>  
  length(a) = length(b) AND  
  (FORALL(i:INDEX): i < length(a) => get(a,i) = get(b,i));
```



-
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A Program Verification

Verification of the following Hoare triple:

$$\{olda = a \wedge oldx = x \wedge n = |a| \wedge i = 0 \wedge r = -1\}$$

while $i < n \wedge r = -1$ **do**

if $a[i] = x$

then $r := i$

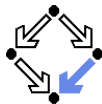
else $i := i + 1$

$$\{a = olda \wedge x = oldx \wedge$$

$$((r = -1 \wedge \forall i : 0 \leq i < |a| \Rightarrow a[i] \neq x) \vee$$

$$(0 \leq r < |a| \wedge a[r] = x \wedge \forall i : 0 \leq i < r \Rightarrow a[i] \neq x))\}$$

Find the smallest index r of an occurrence of value x in array a ($r = -1$, if x does not occur in a).



The Verification Conditions

$A : \Leftrightarrow \text{Input} \Rightarrow \text{Invariant}$

$B_1 : \Leftrightarrow \text{Invariant} \wedge i < n \wedge r = -1 \wedge a[i] = x \Rightarrow \text{Invariant}[i/r]$

$B_2 : \Leftrightarrow \text{Invariant} \wedge i < n \wedge r = -1 \wedge a[i] \neq x \Rightarrow \text{Invariant}[i + 1/i]$

$C : \Leftrightarrow \text{Invariant} \wedge \neg(i < n \wedge r = -1) \Rightarrow \text{Output}$

$\text{Input} : \Leftrightarrow \text{olda} = a \wedge \text{oldx} = x \wedge n = \text{length}(a) \wedge i = 0 \wedge r = -1$

$\text{Output} : \Leftrightarrow a = \text{olda} \wedge x = \text{oldx} \wedge$

$((r = -1 \wedge \forall i : 0 \leq i < \text{length}(a) \Rightarrow a[i] \neq x) \vee$

$(0 \leq r < \text{length}(a) \wedge a[r] = x \wedge \forall i : 0 \leq i < r \Rightarrow a[i] \neq x))$

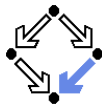
$\text{Invariant} : \Leftrightarrow \text{olda} = a \wedge \text{oldx} = x \wedge n = \text{length}(a) \wedge$

$0 \leq i \leq n \wedge \forall j : 0 \leq j < i \Rightarrow a[j] \neq x \wedge$

$(r = -1 \vee (r = i \wedge i < n \wedge a[r] = x))$

The verification conditions A, B_1, B_2, C have to be proved.

The Verification Conditions



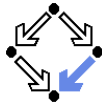
```
newcontext      Input: BOOLEAN = olda = a AND oldx = x AND
  "linsearch";      n = length(a) AND i = 0 AND r = -1;

% declaration      Output: BOOLEAN = a = olda AND
% of arrays        ((r = -1 AND
...                (FORALL(j:NAT): j < length(a) =>
                    get(a,j) /= x)) OR
a: ARR;           (0 <= r AND r < length(a) AND get(a,r) = x AND
olda: ARR;        (FORALL(j:NAT):
x: ELEM;          j < r => get(a,j) /= x)));
oldx: ELEM;

i: NAT;           Invariant: (ARR, ELEM, NAT, NAT, INT) -> BOOLEAN =
n: NAT;           LAMBDA(a: ARR, x: ELEM, i: NAT, n: NAT, r: INT):
r: INT;           olda = a AND oldx = x AND
                  n = length(a) AND i <= n AND
                  (FORALL(j:NAT): j < i => get(a,j) /= x) AND
                  (r = -1 OR (r = i AND i < n AND get(a,r) = x));
...

```

The Verification Conditions (Contd)



...

A: FORMULA

Input \Rightarrow Invariant(a, x, i, n, r);

B1: FORMULA

Invariant(a, x, i, n, r) AND $i < n$ AND $r = -1$ AND $\text{get}(a,i) = x$
 \Rightarrow Invariant(a,x,i,n,i);

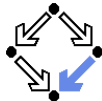
B2: FORMULA

Invariant(a, x, i, n, r) AND $i < n$ AND $r = -1$ AND $\text{get}(a,i) \neq x$
 \Rightarrow Invariant(a,x,i+1,n,r);

C: FORMULA

Invariant(a, x, i, n, r) AND NOT($i < n$ AND $r = -1$)
 \Rightarrow Output;

The Proofs



A: [bca]: expand Input, Invariant
[fuo]: scatter
[bxg]: proved (CVCL)

(2 user actions)

B1: [p1b]: expand Invariant
[lf6]: proved (CVCL)

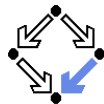
(1 user action)

B2: [q1b]: expand Invariant in 6kv
[slx]: scatter
[a1y]: auto
[cch]: proved (CVCL)
[b1y]: proved (CVCL)
[c1y]: proved (CVCL)
[d1y]: proved (CVCL)
[e1y]: proved (CVCL)

(3 user actions)

C: [dca]: expand Invariant, Output in zfg
[tvy]: scatter
[dcu]: auto
[t4c]: proved (CVCL)
[ecu]: split pkg
[kel]: proved (CVCL)
[lel]: scatter
[lvn]: auto
[lap]: proved (CVCL)
[fcu]: auto
[blt]: proved (CVCL)
[gcu]: proved (CVCL)

(6 user actions)



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1. An Overview of the RISC ProofNavigator
 2. Specifying Arrays
 3. Verifying the Linear Search Algorithm
 - 4. Conclusions**

Conclusions

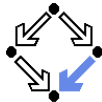


So what does this experience show us?

- Parts of a verification can be handled quite automatically:
 - Top-down proof decomposition.
 - Propositional logic reasoning.
 - Equality reasoning.
 - Linear arithmetic.
- Manual control for crucial “creative steps”
 - Expansion of definitions.
 - Proof cuts by assumptions/case distinctions.
 - Application of additional lemmas.
 - Instantiation of quantified formulas.

Proving assistants can do the essentially simple but usually tedious parts of the proof; the human nevertheless has to provide the creative insight.

Popular Proving Assistants



- **PVS:** <http://pvs.csl.sri.com>
 - SRI (Software Research Institute) International, Menlo Park, CA.
 - Integrated environment for developing and analyzing formal specs.
 - Core system is implemented in Common Lisp.
 - Emacs-based frontend with Tcl/Tk-based GUI extensions.
- **Isabelle/HOL:** <http://isabelle.in.tum.de>
 - University of Cambridge and Technical University Munich.
 - Isabelle: generic theorem proving environment (aka “proof assistant”).
 - Isabelle/HOL: instance that uses higher order logic as framework.
 - Decisions procedures, tactics for interactive proof development.
- **Coq:** <http://coq.inria.fr>
 - LogiCal project, INRIA, France.
 - Formal proof management system (aka “proof assistant”).
 - “Calculus of inductive constructions” as logical framework.
 - Decision procedures, tactics support for interactive proof development.