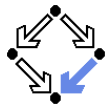


# Model-based Specifications in Larch/C++

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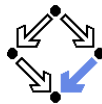


# Model-based Specifications

C.A.R. Hoare: "Proof of Correctness of Data Representations" (1972).

- Verification of abstract datatype implementations.
  - Complements pre/post-condition reasoning about computations.
- Specification uses abstraction function  $\mathcal{A} : C \rightarrow A$ .
  - Maps concrete representations (objects of type Stack) to abstract values (mathematical "stacks").
  - Client of an ADT can reason about its operations without actually knowing its implementation.
- Verification uses inverse concretization function  $\mathcal{C} : A \rightarrow \mathbb{P}(C)$ .
  - Maps abstract values to (sets of) concrete values.
    - $\forall c \in C : c \in \mathcal{C}(\mathcal{A}(c))$ .
    - $\forall a \in A, c \in \mathcal{C}(a) : \mathcal{A}(c) = a$ .
  - Implementation of ADT must prove that its operations satisfy the properties expressed in the specification.

## Example



```
interface Stack { void push(Elem e); Elem pop(); }
```

```

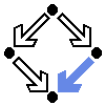
{A(s) = S}
s.push(e);
{A(s) = push(S, A(e))}

{A(s) = S ∧ ¬isEmpty(S)}
e = s.pop();
{A(e) = top(S) ∧ A(s) = pop(S)}

```

Pre/post-conditions in terms of abstract mathematical values.

## Example (Contd)



```

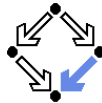
class ArrayStack implements Stack
{
  Elem[] array; int n;
  ...
}

```

- $\mathcal{A} : \text{ArrayStack} \rightarrow \text{Stack}$ .
  - $\mathcal{A}(\text{array}, n) := (\text{informal sketch})$   
 $\text{push}(\dots \text{push}(\text{empty}, \mathcal{A}(\text{array}[0])) \dots, \mathcal{A}(\text{array}[n-1]))$ .
- $\mathcal{C} : \text{Stack} \rightarrow \mathbb{P}(\text{ArrayStack})$ .
  - $\mathcal{C}(\text{empty}) := \{ \langle \text{array}, 0 \rangle \mid \text{Elem}[] \text{ array} \}$ .
  - $\mathcal{C}(\text{push}(s, e)) :=$   
 $\{ \langle \text{array}, l+1 \rangle : \exists a. \langle a, l \rangle \in \mathcal{C}(s) \wedge$   
 $\forall 0 \leq i < l. \text{array}[i] = a[i] \wedge \text{array}[l] = \mathcal{C}(e) \}$

Must prove that  $\mathcal{C}$  is inverse of  $\mathcal{A}$ .

## Example (Contd'2)



```
class ArrayStack { ... void push(Elem e) { body } ... }
```

```
{ $\mathcal{A}(\text{array}, n) = S$ } body { $\mathcal{A}(\text{array}, n) = \text{push}(S, \mathcal{A}(e))$ }
```

```
{ $\langle \text{array}, n \rangle \in \mathcal{C}(S)$ } body { $\mathcal{A}(\text{array}, n) = \text{push}(S, \mathcal{A}(e))$ }
```

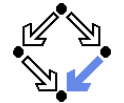
- Case  $S = \text{empty}$ :

```
{ $n = 0$ } body {...}
```

- Case  $S = \text{push}(s, e)$ :

```
{ $\exists l, a . n = l + 1 \wedge \langle a, l \rangle \in \mathcal{C}(s) \wedge$   
 $\forall 0 \leq i < l . \text{array}[i] = a[i] \wedge \text{array}[l] = \mathcal{C}(e)$ }  
body  
{...}
```

## Model-based Specification Languages

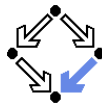


Abstract model specifies vocabulary used in pre/post-conditions.

- **VDM-SL** (Vienna Development Method Specification Language)
  - Started in the IBM laboratory in Vienna in the mid-1970s.
  - (Sort of) functional language to specify models.
- **Z**
  - Started at Oxford University (Hoare and others) in the late 1970s.
  - Set theory and first-order predicate logic to specify models.
- **Larch**: <http://www.sds.lcs.mit.edu/spd/larch>
  - Started at MIT in the late 1970s.
  - Larch Shared Language (LSL) to specify algebraic data types.
  - Several **behavioral interface languages** to specify modules in specific programming languages (including language-specific features).
    - LCL (for C), Larch/Ada, Larch/CLU, Larch/Smalltalk, Larch/C++.

ISO standards for VDM-SL (1996) and for Z (2002).

## Larch/C++



- Behavioral interface specification language for C++.

- Gary T. Leavens, Iowa State University, 1993-1999.
- <http://www.cs.iastate.edu/~leavens/larchc++.html>.

- Shared layer: **LSL traits**.

- Extensible specifications of ADTs.
- Loose interpretation of algebraic specifications.

- Interface layer: **Larch/C++ specification modules**.

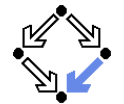
- Specification of C++ classes.
- Includes features dealing with state, aliasing, termination, etc.

- Larch/C++ tools.

- lcpp: parser and type checker.
- lcpp2html: generation of HTML pages.
- LP: prover for reasoning about LSL traits.

Predecessor of the Java Modeling Language (JML).

## Example: Four Sided Figures

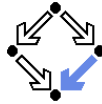


Leavens: "An Overview of Larch/C++ Behavioral Specifications for C++ Modules", 1999.

```
// Quadrilateral.h  
#include "QuadShape.h"  
  
class Quadrilateral : virtual public QuadShape {  
public:  
    Quadrilateral(Vector v1, Vector v2, Vector v3, Vector v4,  
                  Vector pos) throw();  
  
    /* behavior {  
    /* requires isLoop(\<v1,v2,v3,v4\>);  
    /* modifies edges, position;  
    /* ensures liberally edges' = \<v1,v2,v3,v4\> /\ position' = pos;  
    /* }  
};
```

The interface layer.

## Example: Four Sided Figures (Contd'2)



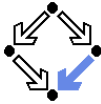
```
// QuadShape.h
#include "Vector.h"
//@ uses FourSidedFigure;
/*@ abstract @*/ class QuadShape {
public:
    //@@ spec Vector edges[4];
    //@@ spec Vector position;
    //@@ invariant isLoop(edges\any);

    virtual Move(const Vector& v) throw();
    //@@ behavior {
    //@@ requires assigned(v, pre);
    //@@ requires redundantly assigned(edges, pre)
    //@@          /\ assigned(position, pre) /\ isLoop(edges^);
    //@@ modifies position;
    //@@ trashes nothing;
    //@@ ensures liberally position' = position^ + v^;
    //@@ ensures redundantly liberally edges' = edges^;
    //@@ example liberally position^ = 0:Vector /\ position' = v^; }

    virtual Vector GetVec(int i)
        const throw();
    //@@ behavior {
    //@@ requires between(1, i, 4);
    //@@ ensures result = edges^[i-1];
    //@@ example i = 1 /\ result = edges^[0]; }

    virtual Vector GetPosition()
        const throw();
    //@@ behavior {
    //@@ ensures result = position^; } };
```

## Example: Four Sided Figures (Contd'3)

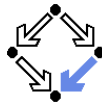


```
% FourSidedFigure.lsl
FourSidedFigure(Scalar): trait
    includes
        PreVector(Scalar, Vector for Vec[T]),
        int, Val_Array(Vector)
    introduces
        isLoop: Arr[Vector] -> Bool
        \<_,_,_,_>:
            Vector, Vector, Vector, Vector
            -> Arr[Vector]
    asserts
        \forall e: Arr[Vector], v1,v2,v3,v4:Vector
        isLoop(e) == (e[0] + e[1] + e[2] + e[3] = 0:Vector);
        \<v1,v2,v3,v4>
        == assign(assign(assign(assign(create(4), 0,v1), 1,v2), 2,v3), 3,v4);

implies
    \forall e: Arr[Vector],
    v1,v2,v3,v4:Vector
    size(\<v1,v2,v3,v4>) == 4;
    (\<v1,v2,v3,v4>)[0] == v1;
    (\<v1,v2,v3,v4>)[1] == v2;
    (\<v1,v2,v3,v4>)[2] == v3;
    (\<v1,v2,v3,v4>)[3] == v4;
    allAllocated(\<v1,v2,v3,v4>);
converts
    isLoop: Arr[Vector] -> Bool,
    \<_,_,_,_>:
        Vector, Vector, Vector, Vector
        -> Arr[Vector]
```

The shared layer.

## Example: Four Sided Figures (Contd'4)



```
% PreVector.lsl
PreVector(T): trait
    assumes RingWithUnit, Abelian(* for \circ),
        TotalOrder, CoerceToReal(T)
    includes PreVectorSpace(T), Real
    introduces
        __ \cdot __: Vec[T], Vec[T] -> T
        length: Vec[T] -> T
    asserts
        \forall u,v,w: Vec[T], a, b: T
        % the inner product is bilinear
        (u + v) \cdot w == (u \cdot w) + (v \cdot w);
        u \cdot (v + w) == (u \cdot v) + (u \cdot w);
        (a * u) \cdot v == a * (u \cdot v);
        (a * u) \cdot v == u \cdot (a * v);

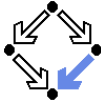
    % ... and is commutative
    u \cdot v == v \cdot u;

    % ... and is positive definite
    (u \cdot u) >= 0;
    (u \cdot u = 0) == (u = 0);

    approximates(length(u),
        sqrt(toReal(u \cdot u)));

    implies
        PreVectorSig(T)
    converts
        __ \cdot __: Vec[T], Vec[T] -> T
```

## Example: Four Sided Figures (Contd'5)



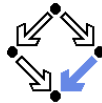
```
% PreVectorSpace.lsl
PreVectorSpace(T): trait
    assumes RingWithUnit, Abelian(* for \circ)
    includes
        AbelianGroup
        (Vec[T] for T, + for \circ,
        0 for unit, - __ for \inv),
        DistributiveRingAction
        (T for M, Vec[T] for T)
    implies
        AC(+ for \circ, Vec[T] for T),
        Idempotent(- __, Vec[T])
        \forall u,v,w: Vec[T], a, b: T
        a * (u + v) == (a * u) + (a * v);
        (a + b) * u == (a * u) + (b * u);
        (a * b) * u == a * (b * u);
        1 * u == u;
        u - v == u + (- v);
        (u + v == u + w) => v = w;
        0 * u == 0:Vec[T];

        -(a * u) == (-a) * u;
        -(a * u) == a * (-u);
        (-a) * (-u) == a * u;
        (a \neq 0 /\ a * u = a * v) =>
            u = v;

    converts
        0: -> Vec[T],
        __+__: Vec[T], Vec[T] -> Vec[T],
        __*__: T, Vec[T] -> Vec[T],
        - __: Vec[T] -> Vec[T],
        __ - __: Vec[T], Vec[T] -> Vec[T]

    PreVectorSig(T): trait
    introduces
        __ + __: Vec[T], Vec[T] -> Vec[T]
        __ * __: T, Vec[T] -> Vec[T]
        0: -> Vec[T]
        - __: Vec[T] -> Vec[T]
        __ - __: Vec[T], Vec[T] -> Vec[T]
        __ \cdot __: Vec[T], Vec[T] -> T
        length: Vec[T] -> T
```

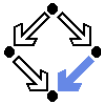
## Example: Four Sided Figures (Contd'6)



```
edsger2!448> lcpp
Usage: lcpp [preprocessor-options] [checker-options] file1.h [file2.lh ...]
The checker-options are:
  --no-verbose    (don't print verbose messages)
  --no-LSL        (don't run the LSL checker)
  --keep-LSL      (keep LSL trait files if they have errors)
The currently understood preprocessor options are:
  -ansi -Dmacro[=defn] -Umacro -Aquestion[(answer)] -nostdinc++ -undef
  -I dir -H dir -include file -imacros file -iprefix prefix
  -iwithprefix dir -idirafter dir
```

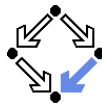
Syntax and type checking; no verification!

## Example: Four Sided Figures (Contd'7)



```
edsger2!447> lcpp Quadrilateral.h
LCPP_builtins is up to date.
Checking Quadrilateral.h ...
Checking trait: Scalar
Finished checking LSL traits
Checking trait: PreVector(Scalar,Vector for Vec[T])
Finished checking LSL traits
NoContainedObjects(Vector) is up to date.
Checking trait: FourSidedFigure
Finished checking LSL traits
NoContainedObjects(Shear) is up to date.
Quadrilateral.h 0 warnings; 0 syntax & 0 semantic errors!
```

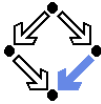
## Proving LSL Properties



LP (the Larch Prover), Release 3.1b (98/06/09) logging to  
'/usr3/Larch/lp3.1b/samples/list1.lpllog' on 18 October 2005 16:18:26.

```
LP0.1.9: declare sorts Element, List
LP0.1.10: declare variables e: Element, x, y, z: List
LP0.1.11: declare operators
null    :                -> List
cons    : Element, List -> List
append  : List, List    -> List
rev     : List           -> List
..
LP0.1.15: assert
sort List generated by null, cons;
append(null, x) = x;
append(cons(e, y), z) = cons(e, append(y, z));
rev(null) = null;
rev(cons(e, y)) = append(rev(y), cons(e, null))
..
```

## Proving LSL Properties (Contd)



LP0.1.22: prove  $\text{rev}(\text{rev}(x)) = x$  by induction

Attempting to prove conjecture theorem.1:  $\text{rev}(\text{rev}(x)) = x$   
Creating subgoals for proof by structural induction on 'x'  
Basis subgoal:

Subgoal 1:  $\text{rev}(\text{rev}(\text{null})) = \text{null}$

Induction constant: xc

Induction hypothesis:

theoremInductHyp.1:  $\text{rev}(\text{rev}(xc)) = xc$

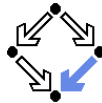
Induction subgoal:

Subgoal 2:  $\text{rev}(\text{rev}(\text{cons}(e, xc))) = \text{cons}(e, xc)$

Attempting to prove level 2 subgoal 1 (basis step) for proof by induction on x  
Level 2 subgoal 1 (basis step) for proof by induction on x  
[] Proved by normalization.

Attempting to prove level 2 subgoal 2 (induction step) for proof by induction  
on x  
Added hypothesis theoremInductHyp.1 to the system.  
Suspending proof of level 2 subgoal 2 (induction step) for proof by induction  
on x

## Proving LSL Properties (Contd'2)



LPO.1.24: % We need a lemma about  $\text{rev}(\text{append}(x, y))$ .  
LPO.1.26: prove  $\text{rev}(\text{append}(x, y)) = \text{append}(\text{rev}(y), \text{rev}(x))$  by induction on  $x$

Attempting to prove level 3 lemma theorem.2:  
 $\text{rev}(\text{append}(x, y)) = \text{append}(\text{rev}(y), \text{rev}(x))$

Creating subgoals for proof by structural induction on 'x'  
Basis subgoal:

Subgoal 1:  $\text{rev}(\text{append}(\text{null}, y)) = \text{append}(\text{rev}(y), \text{rev}(\text{null}))$

Induction constant:  $xc1$

Induction hypothesis:

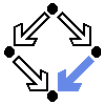
theoremInductHyp.2:  $\text{rev}(\text{append}(xc1, y)) = \text{append}(\text{rev}(y), \text{rev}(xc1))$

Induction subgoal:

Subgoal 2:  $\text{rev}(\text{append}(\text{cons}(e, xc1), y)) = \text{append}(\text{rev}(y), \text{rev}(\text{cons}(e, xc1)))$

Attempting to prove level 4 subgoal 1 (basis step) for proof by induction on  $x$   
Suspending proof of level 4 subgoal 1 (basis step) for proof by induction on  $x$

## Proving LSL Properties (Contd'3)



LPO.1.28: % We need another lemma, which we obtain by generalization.  
LPO.1.30: prove  $\text{append}(x, \text{null}) = x$  by induction

Attempting to prove level 5 lemma theorem.3:  $\text{append}(x, \text{null}) = x$   
Creating subgoals for proof by structural induction on 'x'

Basis subgoal:

Subgoal 1:  $\text{append}(\text{null}, \text{null}) = \text{null}$

Induction constant:  $xc1$

Induction hypothesis:

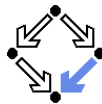
theoremInductHyp.2:  $\text{append}(xc1, \text{null}) = xc1$

Induction subgoal:

Subgoal 2:  $\text{append}(\text{cons}(e, xc1), \text{null}) = \text{cons}(e, xc1)$

Attempting to prove level 6 subgoal 1 (basis step) for proof by induction on  $x$   
Level 6 subgoal 1 (basis step) for proof by induction on  $x$   
[] Proved by normalization.

## Proving LSL Properties (Contd'4)



Attempting to prove level 6 subgoal 2 (induction step) for proof by induction on  $x$

Added hypothesis theoremInductHyp.2 to the system.

Level 6 subgoal 2 (induction step) for proof by induction on  $x$

[] Proved by normalization.

Level 5 lemma theorem.3

[] Proved by structural induction on 'x'.

Attempting to prove level 4 subgoal 1 (basis step) for proof by induction on  $x$

Level 4 subgoal 1 (basis step) for proof by induction on  $x$ :

$\text{rev}(\text{append}(\text{null}, y)) = \text{append}(\text{rev}(y), \text{rev}(\text{null}))$

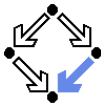
[] Proved by normalization.

Attempting to prove level 4 subgoal 2 (induction step) for proof by induction on  $x$ :  $\text{rev}(\text{append}(\text{cons}(e, xc1), y)) = \text{append}(\text{rev}(y), \text{rev}(\text{cons}(e, xc1)))$

Added hypothesis theoremInductHyp.2 to the system.

Suspending proof of level 4 subgoal 2 (induction step) for proof by induction on  $x$

## Proving LSL Properties (Contd'5)



LPO.1.32: % We need another lemma (the associativity of append)

LPO.1.35: prove  $\text{append}(\text{append}(x, y), z) = \text{append}(x, \text{append}(y, z))$  by induction on  $x$

Attempting to prove level 5 lemma theorem.3:

$\text{append}(\text{append}(x, y), z) = \text{append}(x, \text{append}(y, z))$

Creating subgoals for proof by structural induction on 'x'

Basis subgoal:

Subgoal 1:  $\text{append}(\text{append}(\text{null}, y), z) = \text{append}(\text{null}, \text{append}(y, z))$

Induction constant:  $xc2$

Induction hypothesis:

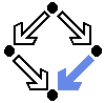
theoremInductHyp.3:  $\text{append}(\text{append}(xc2, y), z) = \text{append}(xc2, \text{append}(y, z))$

Induction subgoal:

Subgoal 2:  $\text{append}(\text{append}(\text{cons}(e, xc2), y), z) = \text{append}(\text{cons}(e, xc2), \text{append}(y, z))$

Attempting to prove level 6 subgoal 1 (basis step) for proof by induction on  $x$   
Level 6 subgoal 1 (basis step) for proof by induction on  $x$   
[] Proved by normalization.

## Proving LSL Properties (Contd'6)



Attempting to prove level 6 subgoal 2 (induction step) for proof by induction on  $x$

Added hypothesis theoremInductHyp.3 to the system.

Level 6 subgoal 2 (induction step) for proof by induction on  $x$

[] Proved by normalization.

Level 5 lemma theorem.3

[] Proved by structural induction on 'x'.

Attempting to prove level 4 subgoal 2 (induction step) for proof by induction on  $x$ :  $\text{rev}(\text{append}(\text{cons}(e, \text{xc1}), y)) = \text{append}(\text{rev}(y), \text{rev}(\text{cons}(e, \text{xc1})))$

Current subgoal:

```
append(append(rev(y), rev(xc1)), cons(e, null))
= append(rev(y), append(rev(xc1), cons(e, null)))
```

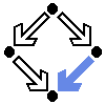
Level 4 subgoal 2 (induction step) for proof by induction on  $x$

[] Proved by normalization.

Level 3 lemma theorem.2:  $\text{rev}(\text{append}(x, y)) = \text{append}(\text{rev}(y), \text{rev}(x))$

[] Proved by structural induction on 'x'.

## Proving LSL Properties (Contd'7)



Attempting to prove level 2 subgoal 2 (induction step) for proof by induction

on  $x$ :  $\text{rev}(\text{rev}(\text{cons}(e, \text{xc}))) = \text{cons}(e, \text{xc})$

Current subgoal:  $\text{rev}(\text{append}(\text{rev}(\text{xc}), \text{cons}(e, \text{null}))) = \text{cons}(e, \text{xc})$

Level 2 subgoal 2 (induction step) for proof by induction on  $x$

[] Proved by normalization.

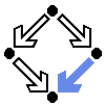
Conjecture theorem.1:  $\text{rev}(\text{rev}(x)) = x$

[] Proved by structural induction on 'x'.

LPO.1.36: qed

All conjectures have been proved.

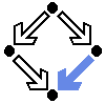
## The Java Modeling Language



- Behavioral interface specification language for Java.
  - Gary T. Leavens et al., Iowa State University, since 1999.
  - <http://www.jmlspecs.org>
- Fully embedded into the Java language.
  - No separation between shared layer and interface layer anymore.
  - All specifications expressed in (an extended version of) Java.
- Considerable community support.
  - jml: syntax and type checking.
  - jmldoc: document generation.
  - JMLEclipse: plugin for the Eclipse IDE.
  - ESC/Java2: extended static checking of JML specifications.

Java programmer needs not learn a new expression language, but distinction between model and representation gets blurred.

## A Stack Model

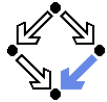


```
public /*@ pure @*/ class IntStackModel
{
    // IntStackModel() is default constructor

    /*@ public model boolean isempty();
    /*@ public model IntStackModel push(int e);
    /*@ public model int top();
    /*@ public model IntStackModel pop();

    /*@ public invariant
    @ (\forall IntStackModel s, s2; s != null;
    @   (\forall int e, e2;
    @     !new IntStackModel().equals(s.push(e)) &&
    @     (s.push(e).equals(s2.push(e2)) ==> s.equals(s2) && e == e2) &&
    @     new IntStackModel().isempty() &&
    @     !s.push(e).isempty() &&
    @     e == s.push(e).top() &&
    @     s.equals(s.push(e).pop()));
    @*/
}
```

# A Stack Implementation



```
public class IntStack // "IntStack.jml"
{
  /*@ public model
   @ non_null IntStackModel stackM;
   @ public initially stackM.isempty();
   @
   @ represents stackM <- toModel();
   @ public model
   @ pure IntStackModel toModel(); @*/

  /*@ public normal_behavior
   @ assignable stackM;
   @ ensures stackM.isempty(); @*/
  public IntStack();

  /*@ public normal_behavior
   @ assignable \nothing;
   @ ensures \result <==> stackM.isempty(); @*/
  public /*@ pure @*/ boolean isempty();

  /*@ public normal_behavior
   @ assignable stackM;
   @ ensures stackM ==
   @   \old(stackM.push(e)); @*/
  public void push(int e);

  /*@ public normal_behavior
   @ requires !stackM.isempty();
   @ assignable stackM;
   @ ensures \result ==
   @   \old(stackM.top())
   @   && stackM ==
   @   \old(stackM.pop()); @*/
  public int pop(int e);
}
```

See course on “Formal Methods in Software Development”.