Fundamentals of Numerical Analysis and Symbolic Computation
Exercise (January 14)

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The result is to be submitted by the deadline stated above via the Moodle interface of the course "Computer-Based Working Environments" as a PDF file (with an appropriate cover page and a section for each exercise, handwritten results are also acceptable but have to be scanned and included in the file).

Exercise 1

Verify formally the correctness of the Hoare Triple

$$\{a \in \mathbb{N} \wedge a = olda \wedge b \in \mathbb{N} \wedge b = oldb \wedge c \in \mathbb{N} \wedge c = oldc\}$$

```
if (b < a) a = b;
if (c < a) a = c;
```

$$\{(a = olda \vee a = oldb \vee a = oldc) \wedge (a \leq olda \vee b \leq oldb \vee c \leq oldc)\}$$

which specifies a program that computes the minimum of three natural numbers a, b, c and stores it in a.

Show in detail the derivation of the verification conditions (either by the basic Hoare calculus or by application of weakest precondition reasoning).

Exercise 2

Verify formally the *total* correctness of the Hoare Triple

$$\{n \in \mathbb{N} \wedge n = oldn \wedge a : \mathbb{N} \to \mathbb{N} \wedge a = olda\}$$

```
s = 0;
i = 0;
while (i <= n)
{
  s = s + a[i];
  i = i+1;
}
```

$$\{n = oldn \wedge a = olda \wedge s \in \mathbb{N} \wedge s = \textstyle\sum_{i=0}^{n} a[i]\}$$

which specifies a program for the computation of the sum s of the array values $a[0] \ldots a[n]$. Explicitly state the invariant and termination term you use in the verification.

Exercise 3

Verify formally the *total* correctness of the Hoare Triple

$$\{x \in \mathbb{N} \wedge x = oldx\}$$

```
y = 0;
z = 1;
while (z <= x)
{
  y = y+1;
  z = z+2*y+1;
}
```

$$\{x = oldx \wedge y \in \mathbb{N} \wedge y^2 \leq x < (y+1)^2\}$$

which specifies a program for the computation of the truncated square root y of a natural number x. Explicitly state the invariant and termination term you use in the verification.

Exercise 4

Verify formally the *total* correctness of the Hoare Triple

$$\{n \in \mathbb{N} \wedge n = oldn \wedge a : \mathbb{N} \to \mathbb{N} \wedge a = olda \wedge \\ x \in \mathbb{N} \wedge x = oldx \wedge y \in \mathbb{N} \wedge y = oldx\}$$

```
i = 0;
while (i < n)
{
  if (a[i] == x) a[i] = y;
  i = i+1;
}
```

$$\{n = oldn \wedge x = oldx \wedge y = oldy \\ \forall i \in \mathbb{N} : i < n \Rightarrow \\ (olda[i] = x \Rightarrow a[i] = y) \wedge \\ (olda[i] \neq x \Rightarrow a[i] = olda[i])\}$$

which specifies a program for replacing in an array a all occurrences of x by y. Explicitly state the invariant and termination term you use in the verification.